

***UNIVERSITY OF OSLO***  
***DEPARTMENT OF ECONOMICS***

Exam: ECON4310 – Consumption, investment and pensions, fall 2003

Date of exam: Friday, January 16, 2004

Time for exam: 9 a.m. – 12 noon

The problem set covers 4 pages

Resources allowed:

- Calculator

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

# 4310 – Consumption, Investment and Pensions

Exam, 9:00-12:00 January 16, 2004

Before you start, please read the following:

- You are allowed to use a calculator (“lommekalkulator”) on the exam.
- You can answer in either English or Norwegian.
- Answer all questions and write brief and concise answers!
- Good style will not matter for grades, but please write clearly.
- Good luck!

## 1 Question 1

### Land and asset pricing.

Suppose the utility function is given by

$$u_t^h = \log c_t^h(t) + \frac{1}{2} \log (c_t^h(t+1))$$

and that endowments are  $\omega_t^h = [3, 4]$ . The population is constant at  $N = 10$  and there exists two assets, private lending (which pays an interest rate  $r(t)$ ) and  $A = 10$  units of land, which pays a dividend of  $d$  per unit of land each period. As demonstrated in the textbook (you do not have to derive this), in this economy individuals have a savings function given by

$$s(r(t)) = 1 - \frac{8}{3} \frac{1}{r(t)}.$$

1. Write down the equilibrium conditions for a time  $t$  temporary equilibrium. and illustrate in a diagram how one can find temporary equilibria, given expectations about the future price of land.
2. Write down equilibrium conditions for a perfect foresight competitive equilibrium. Why are there more temporary equilibria than perfect foresight equilibria?
3. Assume that  $d = 1$  and show that the stationary competitive equilibrium price of land and the associated interest rate are given by  $p = 1/3$  and  $r = 4$ .
4. Show (using diagrammatic analysis) that there cannot be other competitive equilibria.
5. Is the competitive equilibrium Pareto optimal? Please explain.

## 2 Question 2

### Expectations hypothesis.

Suppose the prices of one-period bonds and two-period (zero coupon) bonds in period  $t$ , i.e. the “yield curve”, are given by  $p_1(t) = \frac{9}{10}$  and  $p_2(t) = \frac{9}{11}$ , respectively.

1. Explain why in equilibrium it must be that the interest rate is given by the price of the one-period bond,  $r(t) = \frac{1}{p_1(t)}$ .
2. Explain why perfect foresight/rational expectations can be used to forecast  $r(t+1)$ , the interest rate in period  $t+1$  (where the interest rate in period  $t$  is implied by  $p_1(t)$ , the price of a one-period bond in period  $t$ ).
3. Compute the forecasted interest rate in period  $t+1$ .

## 3 Question 3

### Precautionary savings.

Consider the savings-problem of an agent who lives for two periods and has an endowment of  $\omega_1 = 1$  when young. The endowment when old,  $\omega_2$ , is stochastic with

$$\begin{aligned} E(\omega_2) &= 1 \\ \text{var}(\omega_2) &= \sigma^2. \end{aligned}$$

The agent maximizes

$$\max_{c_1, c_2} \{u(c_1) + u(c_2)\},$$

subject to

$$\begin{aligned} c_1 &= \omega_1 - s \\ c_2 &= \omega_2 + s. \end{aligned}$$

1. Suppose, first, that the utility function is linear-quadratic, so

$$u(c) = c - \frac{a}{2}c^2,$$

where  $a \in (0, 1)$ . Compute the optimal savings  $s$ . Would the result change if one instead considered an alternative case with  $\text{var}(\omega_2) = 0$ ?

2. Suppose instead that  $\omega_1 = 2$  and  $\omega_2 = 0$ , so there is no risk in the second-period endowment.

- (a) Compute the new level of savings  $s$ . Explain why the optimal consumption-path is the same as the one above.

- (b) Suppose instead that  $a = 0$ , so that utility is linear. Explain why the agent does not care any longer about consumption smoothing.
  - (c) Explain what feature of the utility function  $u$  that imply consumption smoothing.
3. Consider instead a case with the following alternative “constant relative risk-aversion” utility function:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma},$$

where  $\gamma > 0$ . Go back to the case with  $E(\omega_2) = 1$  and  $var(\omega_2) > 0$ . Prove that savings will increase relative to the quadratic utility case above.

4. Give the intuition for why savings are larger under the second utility function (with constant relative risk-aversion). In particular, explain what feature of the utility function that matters.