UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: ECON4310 - Consumption, investment and pensions, fall 2003

Date of exam: Wednesday, December 3rd 2003

Time for exam: 9 a.m. – 12:00

The problem set covers 4 pages (including this)

Resources allowed:

- Calculator

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

4310 – Consumption, Investment and Pensions

Exam, 9:00-12:00 December 3, 2003

Before you start, please read the following:

- You are allowed to use a calculator ("lommekalkulator") on the exam.
- You can answer in either English or Norwegian.
- Answer all questions and write brief and concise answers!
- Good style will not matter for grades, but please write clearly.
- Good luck!

1 Question 1

Suppose the utility function is given by

$$u_t^h = c_t^h(t) * c_t^h(t+1)$$

and that endowments are equal to $\omega_t^h = [3, 1]$. The population is constant; $N(t+1) = N(t)$ for all t. There exists one asset; private lending which pays an interest rate $r(t)$. As demonstrated in the textbook (you do **not** have to derive this), in this economy individuals have a savings function given by

$$s_t^h(r(t)) = \frac{3 - \tau_t(t)}{2} - \frac{1 - \tau_t(t+1)}{2r(t)},$$

where $\tau_t(j)$ are taxes. Assume, first, that taxes are zero.

1. Write down the equilibrium condition for this economy, and show that the equilibrium interest rate is $r(t) = 1/3$.
2. Is the economy Pareto optimal? Please explain.
3. Explain why a tax and transfer scheme where each period the young give one unit to the old would be Pareto improving.
4. Suppose, alternatively, that parents care about their children and that all parents, when they are old, give their children a bequest. What would be the effect of the tax and transfer scheme described above?

2 Question 2

Asset pricing: Assume that the utility function is

$$u_t^h\left(c_t^h(t+1)\right) = \log\left(c_t^h(t)\right) + \beta \cdot \log\left(c_t^h(t+1)\right).$$

and that endowments are constant over time; $\omega_t^h = [\omega_1, \omega_2]$. The population is constant at $N(t) = N$, and there exists two assets, private lending (which pays an interest rate $r(t)$) and A units of land, which pays a stochastic crop $d(t) = 1 + \varepsilon(t+1)$ per unit of land in period t. The crop has a variance $var(d(t)) = \sigma^2$ and the price of land is denoted $p(t)$.

1. Write down the equilibrium conditions for this economy and give some brief intuition for these.

2. During the buildup to the Iraq war in the fall of 2002, newspapers were filled with articles indicating that "aggregate uncertainty" has increased. Suppose this can be interpreted as an increase in the risk without affecting the expected crop, i.e. σ increasing while keeping $E\{d(t)\} = 1$. **Explain what happens to the expected excess return in equilibrium as σ increases** (where excess return is return on land minus the the return on lending). Please be formal, using material learnt in class. **How can this explain why world stock prices fell sharply during that period and then increased after May 2003 – the "end of major combat".**

3. Suppose one small island is discovered, and that each old agent receives a negligible amount of this new type of land. In period t, this new land yields $f(t)$ units of fruit per unit of land, where $E\{f(t)\} = 1$. Assume that yield on the different types of land are positively correlated, i.e. $corr(f(t), d(t)) > 0$. Explain how the expected return on the new type of land must be, relative to the safe interest rate and to the expected return on old land. Please use both intuitive arguments AND formal arguments (stuff learnt in class).

3 Question 3

Labor supply: Consider the savings and labor effort problem of an agent who lives for two periods and earns a wage of w_1 when young and w_2 when old. The interest rate is r and the agent maximizes

$$\max_{c_1, c_2, h_1, h_2} \{u(c_1, h_1) + \beta u(c_2, h_2)\},$$

subject to

$$c_1 + \frac{c_2}{r} \leq w_1 h_1 + \frac{w_2 h_2}{r},$$

where c_t is consumption and h_t is labor supply. Assume that the utility function is given by

$$u(c,h) = \frac{c^{1-\gamma}}{1-\gamma} + \frac{(1-h)^{1-\sigma}}{1-\sigma}.$$

1. Suppose, first, that $w_2 = 0$, so that individuals work only when young. What is the effect of the wage rate on labor supply? Is it the workers with high wages or the workers with low wages who work more?

2. Over the last 100 years, wages have increased a lot, while labor effort has declined (on average, we have substantially more leisure than our grandparents). What does this tell us about the parameters of the utility function above?

3. Suppose now that individuals work in both periods of life. All workers earn the same wage when young ($w_1 = 1$). However, when old, some workers have a steep upwards age-profile of wages ($w_2 > 1$), while others have a decreasing age-profile of wages ($w_2 < 1$). What will their age-profile of labor supply look like?