

UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam: ECON4310 – Consumption, investment and pensions

Date of exam: Wednesday, January 12, 2005

Time for exam: 9:00 a.m. – 12:00 noon

The problem set covers 4 pages (including cover page)

Resources allowed:

- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

4310 – Consumption, Investment and Pensions

Exam, 9:00-12:00 January 12, 2005

Before you start, please read the following:

- You are allowed to use a calculator (“lommekalkulator”) on the exam.
- You can answer in either English or Norwegian.
- Answer all questions and write brief and concise answers!
- Good style will not matter for grades, but please write clearly.
- Good luck!

1 Question 1: debt and land (30%)

Suppose the utility function is given by

$$u_t^h = \log c_t^h(t) + \frac{1}{2} \log (c_t^h(t+1))$$

and that endowments are $\omega_t^h = [3, 4]$. The population is constant at $N = 10$ and there exists two assets, private lending (which pays an interest rate $r(t)$) and $A = 10$ units of land, which pays a dividend of $d = 1$ per unit of land each period. As demonstrated in the textbook (you do **not** have to derive this), in this economy individuals have a savings function given by

$$s(r(t)) = 1 - \frac{8}{3} \frac{1}{r(t)}.$$

1. Show that the stationary competitive equilibrium price of land and the associated interest rate are given by $p = 1/3$ and $r = 4$.
2. Show (using diagrammatic analysis) that there cannot be other competitive equilibria.
3. Suppose a government wants to give a lump-sum transfer of 1 unit of consumption good to all agents alive in period t . Moreover, the government contemplates financing this by issuing an appropriate amount of 1-period bonds, but worries about how to pay back the debt next period.

- (a) As an advisor to the finance minister, you try to figure out if it is possible to pay back by simply issuing new debt next period (i.e. roll over the debt next period). Can you come up with a general condition (on dividends or interest rates and growth rates of population and endowments) that ensures that the initial transfer can be financed by rolling over debt indefinitely, without ever taxing future generations? Does this condition hold in the example above?
- (b) Suppose the government decides to finance the transfers in period t by issuing one-period debt in period t and to repay the debt by taxing the old in period $t+1$. Please explain how the same allocation can be achieved by an appropriate mix of taxes and transfers, but without issuing any debt.

2 Question 2: real business cycles (45%)

Consider a standard Real Business Cycle economy. *NOTE: you can answer the questions below without using details of the economy – the details are meant as a way to fix ideas.*

The economy consisting of a large number of identical, price taking firms and a large number of identical, price taking infinitely lived households. Output is given by a Cobb-Douglas function:

$$Y_t = e^{z_t} \cdot K_t^\alpha H_t^{1-\alpha} \quad (1)$$

where T_t is output, K_t is capital stock, and H_t is labor supply. The technology shock evolves according to

$$z_t = \rho z_{t-1} + \varepsilon_t \quad (2)$$

where ε_t is i.i.d. normally distributed disturbance, and capital evolves according to

$$K_{t+1} = (1 - \delta) K_t + I_t,$$

where $\delta < 1$ is the annual depreciation rate and I_t is investment. Output is used for investment and consumption C_t :

$$Y_t = C_t + I_t.$$

Agents have standard preferences over consumption and leisure:

$$u(c, h) = \log c + \log(1 - h)$$

1. Assume first that there are no friction on the labor market, i.e. that all agents are employed and can work as many hours as they like, given a market-clearing wage w_t . How well does the model (without frictions in the labor market) explain empirical data and, in particular, labor market data (fluctuations in aggregate hours worked)?

2. Show how the performance can be improved by introducing a friction in the labor market, namely indivisibility in labor supply (that agents can work either h_0 hours or not at all), combined with complete markets (i.e., lotteries over employment).
3. This model has unemployment in equilibrium. Would stabilization policies (trying to minimize fluctuations in unemployment) be welfare improving in this economy? Please explain.
4. Discuss whether or not this is a good model of aggregate labor supply and unemployment, depending on whether individual unemployment spells are long-lasting or short-lasting.

3 Question 3 (25%)

Consider an economy with constant population, $N(t) = N$ where agents have the following utility function:

$$u_t^h = \log c_t^h(t) + \log(c_t^h(t+1))$$

The endowment of young agents is always $\omega_t(t) = 1$, while the endowment of an old agents is uncertain - half of the old get $\omega_t^h(t+1) = 1/2$ and the others get $\omega_t^h(t+1) = 3/2$, and an individual's outcome is not known at birth:

$$\omega_t(t+1) = \begin{cases} 1/2 & \text{with 50\% probability} \\ 3/2 & \text{with 50\% probability} \end{cases}.$$

There is only one asset - private lending - and no insurance against the endowment risk. Note that there is no aggregate risk (given that there are, say, equally many with high and low second-period endowment).

1. **Write down the first order condition of the agents in order to solve their optimal savings problem.**
2. **Can you express the equilibrium conditions for this economy?**
3. **Compute the competitive equilibrium and the interest rate of the economy.**
4. **Explain why agents would like to buy insurance against the endowment risk when old.**