UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: ECON4310 – Consumption, investment and pensions, autumn 2004

Date of exam: Thursday, November 25, 2004

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 4 pages (including cover page)

Resources allowed:
- Calculator

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

4310 – Consumption, Investment and Pensions

Exam, 14:30-17:30 November 25, 2004

Before you start, please read the following:

- You are allowed to use a calculator ("lommekalkulator") on the exam.
- You can answer in either English or Norwegian.
- Answer all questions and write brief and concise answers!
- Good style will not matter for grades, but please write clearly.
- Good luck!

1 Question 1: asset pricing (30%)

Consider an economy populated by agents who live for two periods. Their preferences are given by

$$u_t^h = u\left(c_t^h(t)\right) + u\left(c_t^h(t+1)\right)$$

where the utility function $u(.)$ is concave and monotone increasing (so $u' > 0$ and $u'' < 0$). Endowments are given by $\omega = [2, 1]$, i.e. two units when young and one when old. The population is constant, $N(t) = N(t-1) = 10$ and there is $A = 10$ units of land, initially held by the old. The yield per unit of land in period t is $d(t) = 0$.

1. Write down the equilibrium conditions.
2. Show that there is a stationary equilibrium with land having a positive value. What is the interest rate and the price of land in this case?
3. Will there be other equilibria? Please explain (state conditions but do not waste time on trying to solve!)
4. Discuss briefly how this simple theory can explain "bubbles" in the asset markets (i.e. that an asset that never pays a dividend can still have a positive price in equilibrium). Is such bubble socially good or socially bad in this economy?
5. Suppose that future dividends suddenly become risky. In particular, assume that $d(t) = 1 + \varepsilon_t$, where ε is i.i.d. (yield tomorrow is independent of yield today or earlier). How do the equilibrium conditions change?

6. Describe the dynamics of the price of land in equilibrium: will it be constant over time or fluctuate?

7. How is the expected return on land relative to the return on private lending?

2 Question 2: real business cycles (45%)

Consider a standard Real Business Cycle economy. *NOTE: you can answer the questions below without using details of the economy – the details are meant as a way to fix ideas.*

The economy consisting of a large number of identical, price taking firms and a large number of identical, price taking infinitely lived households. Output is given by a Cobb-Douglas function:

$$Y_t = e^{z_t} \cdot K_t^{\alpha} H_t^{1-\alpha} \tag{1}$$

where T_t is output, K_t is capital stock, and H_t is labor supply. The technology shock evolves according to

$$z_t = \rho z_{t-1} + \varepsilon_t \tag{2}$$

where ε_t is i.i.d. normally distributed disturbance, and capital evolves according to

$$K_{t+1} = (1 - \delta) K_t + I_t,$$

where $\delta < 1$ is the annual depreciation rate and I_t is investment. Output is used for investment and consumption C_t:

$$Y_t = C_t + I_t.$$

There are no frictions in the labor market and agents have standard preferences over consumption and leisure:

$$u(c, h) = \log c + \log(1 - h)$$

1. Explain *briefly* why in a competitive equilibrium prices (rental rates w_t and $r_t + \delta$) should equal marginal productivity of labor and capital.

2. What happens after a positive technology shock? Please describe and give intuition for the evolution of output Y_t, consumption C_t, investment I_t, capital stock K_t, labor supply H_t, wage rate w_t, and interest rate r_t.

3. In what sense is the propagation of business cycles in this model driven by the exogenous technology process and capital accumulation?

4. Can this model shed light on empirical data?

3 Question 3: labor supply (25%)

Consider the savings and labor effort problem of an agent who lives for two periods and earns a wage of w_1 when young and w_2 when old. The interest rate is r and the agent maximizes

$$\max_{c_1, c_2, h_1, h_2} \left\{ u\left(c_1, h_1\right) + \beta u\left(c_2, h_2\right) \right\},$$

subject to

$$c_1 + \frac{c_2}{r} \leq h_1 + \frac{w_2 h_2}{r},$$

where c_t is consumption and h_t is labor supply, and w_2 is wage rate in period 2 (there is no uncertainty about w_2). Assume that the utility function is given by

$$u\left(c, h\right) = \log c - \frac{h^{1+\sigma}}{1+\sigma}.$$

1. Assume first that $\beta = r$. Explain how labor supply depends on the wage rate in period two (w_2).

2. Assume then that $\beta \neq r$. How does labor supply depend on the interest rate?

3. Suppose that the agent receives a gift when newborn (equal to a), so that the lifetime budget constraint becomes

   $$c_1 + \frac{c_2}{r} \leq a + h_1 + \frac{w_2 h_2}{r}.$$

   How will this affect labor supply?

4. Based on the answers above, consider two countries where wages for young and old ($w = [1, w_2]$) are the same in both countries. One country has no welfare state, and the other has a generous welfare state that provides transfers to the young and lots of public goods. Assume that taxes are zero in both countries (ignoring how the welfare state is financed). How do aggregate labor supply differ in the two countries?