UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: ECON4310 – Consumption, investment and pensions

Date of exam: Wednesday, November 30, 2005 **Grades will be given: January 3, 2006**

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 4 pages (including cover page)

Resources allowed:
- No resources are allwed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

4310 – Consumption, Investment and Pensions

Exam, November 30, 2005

Before you start, please read the following:

- You can answer in either English or Norwegian.
- Answer all questions and write brief and concise answers!
- Good style will not matter for grades, but please write clearly.
- Good luck!

Q 1: Verify the form of the true value fn (25%)

Consider a model economy where the social planner chooses an infinite sequence of consumption and next period's capital stock $\{c_t, k_{t+1}\}_{t=0}^{\infty}$ in order to

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$\begin{aligned} c_t + k_{t+1} &\leq y_t, && \forall t \\ c_t, k_t &\geq 0, && \forall t \\ k_0 &> 0. && \text{given} \end{aligned}$$

Assume the following functional forms

$$\begin{aligned} u(c_t) &= \ln c_t, && \forall t, \sigma > 0 \\ y_t = F(k_t, 1) &= \gamma k_t^{\alpha}, && \forall t, \alpha \in (0,1) \end{aligned}$$

Reformulate this optimization problem as a dynamic programming problem and write up the Bellman equation. Verify that the value function solving the functional equation (i.e. the Bellman equation) is of the following form

$$v(k) = a + b \ln k.$$

Find a and b as functions of the model economy's structural parameters.

Q 2: Business cycles

Part I (20%)

Consider a standard stochastic neoclassical growth economy. *NOTE: you can answer the first four questions below without using details of the economy – the details are meant as a way to fix ideas.*

The economy consisting of a large number of identical, price taking firms and a large number of identical, price taking infinitely lived households. Output is given by a Cobb-Douglas function:

$$Y_t = e^{z_t} \cdot K_t^{\alpha} H_t^{1-\alpha} \tag{1}$$

where T_t is output, K_t is capital stock, and H_t is labor supply. The technology shock evolves according to

$$z_t = \rho z_{t-1} + \varepsilon_t \tag{2}$$

where ε_t is i.i.d. normally distributed disturbance, and capital evolves according to

$$K_{t+1} = (1-\delta) K_t + I_t,$$

where $\delta < 1$ is the annual depreciation rate and I_t is investment. Output is used for investment and consumption C_t:

$$Y_t = C_t + I_t.$$

There are no frictions in the labor market and agents have standard preferences over consumption and leisure:

$$u(c, h) = \log c + \log(1-h)$$

1. Explain *briefly* why in a competitive equilibrium prices (rental rates w_t and $r_t + \delta$) should equal marginal productivity of labor and capital.
2. What happens after a positive technology shock? Please describe and give intuition for the evolution of output Y_t, consumption C_t, investment I_t, capital stock K_t, labor supply H_t, wage rate w_t, and interest rate r_t.
3. In what sense is the propagation of business cycles in this model driven by the exogenous technology process and capital accumulation?

Part II (30%)

1.

4. Reformulate this problem as a dynamic programming problem, i.e. write up the Bellman equation. Describe the problem in terms of (endogenous and exogenous) state variables and control variables.

5. Carefully and concisely describe how you would go along to compute the value function and the decision rules to this stochastic problem using Bellman's method of successive iterations, also called value function iterations.

6. In mathematical terms, define the value function, the decision rules and the laws of motion.

   In words, describe what these three objects tell us.

Q 3: Labor supply (25%)

Consider the savings and labor effort problem of an agent who lives for two periods and earns a wage of w_1 when young and w_2 when old. The interest rate is r and the agent maximizes

$$\max_{c_1, c_2, h_1, h_2} \{u(c_1, h_1) + \beta u(c_2, h_2)\},$$

subject to

$$c_1 + \frac{c_2}{r} \leq h_1 + \frac{w_2 h_2}{r},$$

where c_t is consumption and h_t is labor supply, and w_2 is wage rate in period 2 (there is no uncertainty about w_2). Assume that the utility function is given by

$$u(c, h) = \log c - \frac{h^{1+\sigma}}{1+\sigma}.$$

1. Assume first that $\beta = r$. Explain how labor supply depends on the wage rate in period two (w_2).

2. Assume then that $\beta \neq r$. How does labor supply depend on the interest rate?

3. Suppose that the agent receives a gift when newborn (equal to a), so that the lifetime budget constraint becomes

   $$c_1 + \frac{c_2}{r} \leq a + h_1 + \frac{w_2 h_2}{r}.$$

   How will this affect labor supply?

4. Based on the answers above, consider two countries where wages for young and old ($w = [1, w_2]$) are the same in both countries. One country has no welfare state, and the other has a generous welfare state that provides transfers to the young and lots of public goods. Assume that taxes are zero in both countries (ignoring how the welfare state is financed). How do aggregate labor supply differ in the two countries?