UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: ECON4310 – Consumption, investment and pensions

Date of exam: Wednesday, December 7, 2005 **Grades are given: January 3, 2006**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 6 pages (included cover page)

Resources allowed:

- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

4310 – Consumption, Investment and Pensions

Exam, December 7, 2005

Before you start, please read the following:

- You can answer in either English or Norwegian.
- Answer all questions and write brief and concise answers!
- Allocate time spent on each question wisely.
- Good style will not matter for grades, but please write clearly.
- Good luck!

1 : True or false? (20%)

For each of the statements, true or false, explain why. *Be brief and concise!*

1. If an economy is dynamically inefficient, the introduction of a pay-as-you-go social security system can improve the aggregate welfare.
2. Compared with data, the standard quantitative general equilibrium business cycle model predicts *too high* volatility of labor input.
3. The equity premium puzzle is a *qualitative* puzzle.
4. In the data, for each unit of output produced, about twice as much goes to capital as to labor.
5. In the standard model, the more impatient people are, the higher is the risk-free interest rate.

2 : Intergenerational transfers (20%)

(*Note that in this question some thinking may reduce tedious calculations*)

Consider a household that lives for four periods and which has utility function

$$\ln(c_1) + \ln(c_2) + \ln(c_3) + \ln(c_4)$$

Income in the four periods is: $y_1 = 10$; $y_2 = 40$; $y_3 = 20$ and $y_4 = 10$:

Assume the interest rate is exogenously given and constant equal to 0.

1. Write down the intertemporal budget constraint.
2. Compute the optimal consumption choices $(c_1; c_2; c_3; c_4)$
3. Suppose the household cannot borrow. Now what are the optimal consumption choices?

Now consider two members of the same dynasty that both live for two periods. Children have utility function

$$\ln(c_3) + \ln(c_4)$$

and parents have the utility function

$$\ln(c_1) + \ln(c_2) + v(b)$$

where b are the bequests left to the children and $v(b)$ is the maximal utility children can obtain when given bequests b: Income of parents is $(y_1,\ y_2) = (10;\ 40)$ and that of children is $(y_3,\ y_4) = (20,\ 10)$:

4. Solve the maximization problem of the children to obtain $v(b)$; that is,solve
$$v(b) = \max\{\ln(c_3) + \ln(c_4)\}$$
subject to
$$c_3 + c_4 = y_3 + y_4 + b.$$
5. Use your answer from the previous question to solve the parents' maximization problem. Allow bequests to be negative.
6. Now suppose the government introduces taxes in period 2 of 30 and distributes 30 in lump-sum transfers in period 3. What is the optimal bequest level and consumption allocation now?

3 : Business cycle theory (30%)

Consider the following proto-typical business cycle model.

$$\max_{\{c_t,l_t,k_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right)\right]$$

subject to

$$\begin{aligned}
c_t + i_t &\leq y_t, && \forall t \\
k_{t+1} &= (1-\delta) k_t + i_t && \forall t, \delta \in [0,1] \\
h_t + l_t &= 1, && \forall t \\
c_t, k_t, h_t, l_t &\geq 0, && \forall t \\
k_0 &> 0. && \text{given}
\end{aligned}$$

where the functional forms and law of motion for technology are

$$\begin{aligned}
u\left(c_t, l_t\right) &= \frac{\left(c_t^{\mu} l_t^{1-\mu}\right)^{1-\sigma}}{1-\sigma}, && \forall t, \sigma > 0 \\
y_t &= z_t f\left(k_t, h_t\right) = z_t k_t^{\alpha} h_t^{1-\alpha}, && \forall t, \alpha \in (0,1) \\
z_{t+1} &= \rho z_t + (1-\rho)\,\bar{z} + \omega_{t+1}, && \forall t, \rho \in [0,1], \bar{z} = 1
\end{aligned}$$

and $\{\omega\}_{t=0}^{\infty}$ is a white noise process. The process for z_t can be approximated by a finite-stage Markov process.

1. Write down a representative individual's Bellman equation. What are the individual's control variable(s) and endogenous and exogenous state variables(s) at time $t-1$, t, and $t+1$.

2. Carefully and concisely describe how you would go along to compute the value function and the decision rules to this stochastic problem using Bellman's method of successive iterations, also called value function iterations.

3. Carefully and concisely describe how you would then simulate the economy so as to generate the statistics of business cycle after you have computed the decision rules to this stochastic problem.

4. In what sense is the propagation of business cycles in this model driven by the exogenous technology process and capital accumulation?

4 : Asset pricing (30%)

Consider a stochastic endowment economy which is populated by a large number of identical individuals. All individuals receive the same random endowment stream $\{y_t\}_{t=0}^{\infty}$. There also exists a production technology which produces a random stream $\{d_t\}_{t=0}^{\infty}$ of dividends. The individuals can buy shares (z_t) to this production technology which gives claims to the dividend stream.

Each individual's optimization problem is

$$\max_{\{c_t, z_t\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}$$

subject to her/his period-by-period budget constraint.

$$y_t + z_{t-1}(p_t + d_t) = c_t + p_t z_t.$$

1. Write down a representative individual's Bellman equation and find the representative individual's Euler equation by taking the first order condition(s) and envelope condition(s) of the Bellman equation.

2. Given $\beta < 1$, show that the representative individual's Euler equation can be expressed as

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j}.$$

3. The representative individual has the following utility function

$$u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma}.$$

   (a) Assume that $y_t = 0$ for all t, i.e. pricing the dividend stream is the same as pricing the consumption stream.

   Show that with logarithmic utility, the ratio (price of the consumption stream) / consumption is constant, irrespectively of the distribution of consumption growth.

   (b) Assume that at time t news are published that future consumption (and dividends) will be higher. For $\gamma > 1$, $\gamma = 1$ and $\gamma < 1$ evaluate the effect of this news on the price.

(c) News media often describe it as a puzzle when stock markets decline on good economic news, and often attributes this to expectations of future central bank policies. Given your answer on the previous question, (briefly) make sense of this "puzzle".