UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4310 – Intertemporal macroeconomics**

Date of exam: Thursday, December 6, 2007   **Grades are given: January 4, 2008**

Time for exam: 9:00 a.m. – 12:00 noon

The problem set covers 6 pages (incl. cover sheet)

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

4310 – Intertemporal macroeconomics

Exam, December 6, 2007

Before you start, please read the following:

- You can answer in either English or Norwegian.

- Answer all questions and write brief and concise answers!

- Allocate time spent on each question wisely.

- Good style will not matter for grades, but please write clearly.

- Good luck!

1 : True or false? (20%)

For each of the statements, true or false, explain why. *Be brief and concise!*

1. In business cycle models with indivisible labor supply, as covered in the class, the Frisch elasticity of aggregate labor supply is 0. (5 points)

2. According to historical statistics from Statistics Norway, average annual real aggregate GNP growth between 1865 and 2006 was 1.9%. (5 points)

3. According to growth accounting, the key reason for the slow recovery of U.S. aggregate output during Great Depression is a slow recovery of aggregate Total Factor Productivity. (5 points)

4. When an economy suffers from the problem of dynamic inefficiency, an introduction of a Pay-As-You-Go pension system can overcome this problem. (5 points)

2 : Asset pricing (40%)

Consider a Lucas tree model. The preference of the representative agent is

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}, \ 0 < \beta < 1, \sigma > 0$$

where c_t denotes consumption. Each tree provides a stochastic flow d_t units of nonstorable "fruit", or dividends per period. The economy starts off with each household owning one such tree. Let p_t be the price at time t of a title to all future dividends from a tree.

Let R_t^{-1} be the time-t price for a one-period risk free discounted bond that pays one unit of consumption at at any state in period $t + 1$.

1. Write the representative agent's problem in recursive form, stating explicitly the state variables and control variables in this problem. Find the first order condition. (7 points)

2. Define a recursive competitive equilibrium for this economy. (7 points)

3. Show that with log utility ($\sigma = 1$), the price of a tree is proportional to current period consumption. (5 points)

4. Derive the equilibrium pricing function for risk-free bonds, $R_t^{-1} = R_t^{-1}(d_t)$. Next, suppose d_t follows an i.i.d. process. Under this assumption, would an increase in d_t increase or decrease real interest rate? Explain why. (7 points)

5. We can use this Lucas tree model to price any asset. Suppose d_t follows a two-state Markov Chain. In particular $d_t \in \{d_L, d_H\}$, $d_L < d_H$, where the transition probability follows

$$prob\,(d_{t+1} = d_L \mid d_t = d_L) \ = \ \pi$$
$$prob\,(d_{t+1} = d_H \mid d_t = d_H) \ = \ 1 - \pi$$

Moreover, in this economy households can issue or buy an asset that promise to deliver one unit of fruit next period only when $d_{t+1} = d_L$, and 0 unit of fruit otherwise. Derive the equilibrium price, denoted as q_t, for such an asset (Hint: q_t is contingent on today's state d). Under which current state (d_L or d_H), the price q_t is higher? Explain why. (7 points)

6. Set up the social planner's problem for this Lucas tree economy. Describe concisely in words how you would go long to compute the value function and the decision rules to the social planner's problem using Bellman's method of successive iterations, also called value function iterations (Hint: it's sufficient to show the steps only, without going details to each step). (7 points)

1   3 : Overlapping Generation Model (40%)

Consider an overlapping generations economy where agents live for two period for sure. The economy starts at time 1. Denote the generation born at time t as cohort t. A cohort t agent $(t \geq 1)$ has the preference.

$$U_t = \log(c_{1t}) + \log(c_{2t+1})$$

Each individual has one unit of time to work when young. The individual does not work when old.

Population L_t grows at a rate n so that $L_t = (1+n)L_{t-1}$.

Assume also there is an initial old cohort at time 1 with size $L_0 = 1$.The initial old is endowed with capital stock K_0, which is given exogenously.

A representative firm hire labor and capital in a competitive market and pay wage rate w_t and interest rate r_t. The production function is

$$Y_t = e^{z_t} K_t^\alpha \left(\gamma^t L_t\right)^{1-\alpha}$$

where $\alpha \in (0,1)$, γ is the gross trend growth rate of this economy, and z_t is the deviation of aggregate productivity from the trend growth path. For simplicity, we assume that all agents have perfect foresight about the fluctuation of z_t. Finally, capital stock depreciates fully after one period so that

$$K_{t+1} + C_t = Y_t$$

where C_t is aggregate consumption.

1. Write down the problem for a young agent of cohort t $(t \geq 1)$. Define a competitive equilibrium for this economy. (7 points)

2. Show that this economy can be transform into a stationary sequential economy (no trend growth) by a change of variables. (7 points)

3. Given factor prices, solve for the consumption and saving allocation for a cohort t agent in this stationary economy. What is the level of saving at old age for cohort t, denoted as s_{2t+1}? (7 points)

4. Use the capital market clearing condition to derive the law of motion for capital in this stationary economy. (5 points)

5. Suppose the economy is originally at the balanced growth path, with $z = 0$. At time t, each agents in this economy receives news that at time $t + 1$ there will be an increase in z_{t+1}, that is $z_t = 0, z_{t+1} > 0$. Show how aggregate saving rate at time t will respond to such news. Explain the intuition. (7 points)

6. Suppose now agents value leisure when young. More specifically, assume utility of an agent born at t is

$$U_t = \log(c_{1t}) + \psi \log (1 - l_t) + \log(c_{2t+1})$$

where l_t is the labor supply when young. Again, assume that at time t each agents in this economy receives news that at time $t + 1$ there will be an increase in z_{t+1}, that is $z_t = 0, z_{t+1} > 0$. Verbally describe how labor supply (l_t) for a young agent at time t will respond to such news. Explain the intuition. (7 points)