

UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam/new exam: **ECON4310 – Intertemporal macroeconomics**

Date of exam: Friday, January 9, 2009

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 6 pages (incl. cover sheet)

Resources allowed:

- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

4310 – Intertemporal macroeconomics

Makeup Exam, January, 2009

Before you start, please read the following:

- You can answer in either English or Norwegian.
- Answer all questions and write brief and concise answers!
- Allocate time spent on each question wisely.
- Good style will not matter for grades, but please write clearly.
- Good luck!

1 : True or false? (15%)

For each of the statements, true or false, explain why. *Be brief and concise!*

1. According to growth accounting, the key reason for the slow recovery of U.S. aggregate output during Great Depression is the slow recovery of total factor productivity. (5 points)
2. A dynamically inefficient economy does not satisfy the Modified Golden Rule. (5 points)
3. According to Finn E. Kydland and Edward C. Prescott (1990) “Business cycles: Real facts and a monetary myth”, price level is procyclical and lagging the business cycle with one quarter. (5 points)

2 : The neoclassical growth model (40%)

Consider the following closed model economy: Output (y) is produced from capital (k) and labor (h) input with Cobb-Douglas production technology. The product approach to output in the model economy is

$$y_t = f(k_t, h_t) = k_t^\alpha h_t^{1-\alpha}, \quad \forall t, \alpha \in (0, 1). \quad (1)$$

Markets are competitive, the firms make zero profit, and the factors of production (k and h) paid the value of their marginal products (r and w). The income approach to output in the model economy is

$$y_t = r_t k_t + w_t h_t, \quad \forall t. \quad (2)$$

The expenditure approach to output in the model economy is

$$y_t = c_t + i_t, \quad \forall t, \quad (3)$$

where i is investment and c is both private and government consumption.

The law of motion for capital accumulation is

$$k_{t+1} = (1 - \delta) k_t + i_t \quad \forall t, \delta \in [0, 1]. \quad (4)$$

1. Since the economy is closed, gross investment will be equal to savings. Make the following behavioral assumption: savings is a constant fraction σ of disposable income.

One year a natural catastrophe hits our model economy and destroys 50% of the capital stock. No individuals are hurt and nobody change their behavior. We are interested in knowing how much of the capital stock has been rebuilt after, say, 25 years. Please write pseudo code for a small computer program that answers this question. (18 points)

2. Make the following changes to our model economy:

- (a) The individuals in the economy supply labor inelastically and choose consumption and investment to maximize their welfare which is given by the following utility function

$$\max_{\{c_t, i_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln c_t.$$

- (b) Capital depreciates fully between every two periods:

$$\delta = 1.$$

The first welfare theorem holds so we can solve this problem as a social planner's problem. Why are we interested in solving the social planner's problem? (4 points)

3. Reformulate the social planner's optimization problem as a dynamic programming problem and write up the Bellman equation. Verify that the value function solving the functional equation (i.e. the Bellman equation) is of the following form

$$v(k) = a + b \ln k.$$

Find a and b as functions of the model economy's structural parameters. (18 points)

3 : Overlapping Generations Economy (10 %)

Consider an overlapping generations economy where agents live for two period for sure. Suppose that N_t two-period-lived individuals are born in period t and that $N_t = (1 + n)N_{t-1}$. For simplicity, let utility be logarithmic with no discounting: $U_t = \log(c_{1t}) + \log(c_{2t+1})$. Individuals work only when young, supply one unit of labor inelastically and earn a wage w_t . A firm hires labor and capital in competitive markets. The production function in per-capita form is

$$f(k_t) = k_t^\alpha$$

where $\alpha \in (0, 1)$ and there is no depreciation.

1. Find an individual agent's consumption when young and when old, given factor prices. (5 points)
2. Suppose that there is a one time positive shock $\epsilon > 0$ to population growth so that

$$N_{t-1} = (1 + n) N_{t-2}, N_{t+1} = (1 + n + \epsilon) N_t, N_{t+2} = (1 + n + \epsilon) N_{t+1}$$

Describe in words the effects of such shock on interest rate at period t , $t + 1$ and the final steady state. (5 points)

4 : Real Business Cycle Model with Complete Capital Depreciation (35 %)

Consider a real business cycle model where a planner chooses a sequence of consumption, investment and hours to maximize the following objective

$$\max \sum_{t=0}^{\infty} \beta^t (\log c_t + B \log(1 - h_t)), \quad 0 < \beta < 1, B > 0$$

subject to

$$\begin{aligned} c_t + k_{t+1} &= z_t k_t^\theta h_t^{1-\theta}, \quad \gamma \geq 1, 0 < \theta < 1 \\ 0 &\leq h_t \leq 1 \\ k_0 &\text{ given} \end{aligned}$$

There is no population growth, no technological progress. The random variable z_t is observed at the beginning of time t . Although future z are unknown, the answer to this question will not depend on details of the probability distribution governing these shocks.

1. Derive the Euler Equation associated with this optimization problem and the first order condition for h_t . (5 points)
2. Verify that if the planner invests a constant fraction of total output each period, the Euler equation would be satisfied for each $t \geq 1$ (not just the steady state). That is, show that a constant saving rate each period is optimal ($k_t = \mu z_t k_t^\alpha h_t^{1-\alpha}$ for some μ). What is the optimal saving rate (the value of μ) as a function of α and β ? Explain why the expectation of future technological changes has no effect on current consumption. (15 points)
3. Using the results derived in question (2) in the first order condition for h_t , shows that hours worked is constant over time. Explain why hours worked is constant over time. Does this seems like a reasonable model for studying business cycle? Explain. (15 points)