UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4310 – Intertemporal macroeconomics**

Date of exam: Thursday, November 27, 2008 **Grades are given: December 19, 2008**

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 7 pages (incl. cover sheet)

Resources allowed:

- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

4310 – Intertemporal macroeconomics

Exam, November 27, 2008

Before you start, please read the following:

- You can answer in either English or Norwegian.
- Answer all questions and write brief and concise answers!
- Allocate time spent on each question wisely.
- Good style will not matter for grades, but please write clearly.
- Good luck!

1 : True or false? (20%)

For each of the statements, true or false, explain why. *Be brief and concise!*

1. In business cycle models with indivisible labor supply, as covered in the class, the Frisch elasticity of individual labor supply is infinity. (5 points)
2. According to the stylized facts of U.S. business cycles, hours worker per person and wage rate is highly correlated. (5 points)
3. An economy with a steady-state capital stock satisfying the Modified Golden Rule will always be dynamic efficient. (5 points)
4. The standard stochastic neoclassical growth model generates a volatility of consumption relative to that of output higher than observed in the data. (5 points)

2 : Reduction of the work week (40%)

Consider the following model economy: Individuals maximize the sum of discounted utility derived from consumption and leisure

$$\max_{\{c_t, l_t, i_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

The utility function is given the following parametric form

$$u(c_t, l_t) = \ln c_t + \psi \ln l_t, \qquad \forall t. \tag{1}$$

The economy is closed and that consumption, c_t, is both private and government consumption. The expenditure approach to output in the model economy is

$$y_t = c_t + i_t, \qquad \forall t. \tag{2}$$

Markets are competitive and the firms make zero profit. Output is paid out to labor and capital. The income approach to output in the model economy is

$$y_t = r_t k_t + w_t h_t, \qquad \forall t. \tag{3}$$

Output is produced from capital and labor input with Cobb-Douglas production technology. The product approach to output in the model economy is

$$y_t = f(k_t, h_t) = k_t^{\alpha} h_t^{1-\alpha}, \qquad \forall t, \alpha \in (0, 1). \tag{4}$$

The law of motion for capital accumulation is

$$k_{t+1} = (1 - \delta) k_t + i_t \qquad \forall t, \delta \in [0, 1]. \tag{5}$$

Total time available for an individual can be spent on leisure or supplied in the market. Without loss of generality we normalize total time available to 1

$$h_t + l_t = 1, \qquad \forall t. \tag{6}$$

Consumption, capital stock, hours supplied in the market and leisure are all strictly positive

$$c_t, k_t, h_t, l_t \geq 0, \qquad \forall t. \tag{7}$$

1. The first welfare theorem holds so we can solve this problem as a social planner's problem. Why are we interested in solving the social planner's problem? (2 points)

2. This problem has a recursive structure. Write up the recursive problem. What are the (endogenous / exogenous) state variable(s) and what are the control variable(s)? (4 points)

3. Take the first order condition of the Bellman equation with respect to the control variable(s) and the envelope condition of the Bellman equation with respect to the endogenous state variable(s) to show that the intertemporal optimality condition is

$$\beta R_{t+1} \frac{u_1\left(c_{t+1}, l_{t+1}\right)}{u_1\left(c_t, l_t\right)} = \beta\left(1 + f_1\left(k_{t+1}, h_{t+1}\right) - \delta\right) \frac{c_t}{c_{t+1}} = 1$$

   and the intratemporal optimality condition is

$$w_t \frac{u_1\left(c_t, l_t\right)}{u_2\left(c_t, l_t\right)} = \frac{f_2\left(k_t, h_t\right)}{\psi} \frac{1-h_t}{c_t} = 1.$$

   (5 points)

4. Show that in steady state, the intertemporal optimality condition implies

$$\beta\left(\alpha\left(\frac{k}{y}\right)^{-1} + 1 - \delta\right) = 1$$

   the intratemporal optimality condition implies

$$(1-\alpha)\left(\frac{c}{y}\right)^{-1} = \frac{1-\psi}{\psi} \frac{h}{1-h}$$

   and the law of motion of the capital stock

$$\delta = \frac{i}{k}.$$

Table 1: Calibration: Parameters and moments to match

Parameter	Description	Moment to match
α	Capital's share	Avg. capital's share of output
β	Time preference	Avg. capital-to-output ratio
δ	Depreciation	Avg. investment-to-capital-stock ratio
ψ	Weight on cons.	Avg. consumption-to-output ratio

In order to determine h, we rely on microeconomic evidence. Most studies find that household allocate about one-third of their discretionary time – ie. time not spent sleeping or in personal maintenance activities – to market activities. We use $h = \frac{1}{3}$.

Calibrate α, δ, β and ψ.

Table 2: Calibration: Measured moments

Moment	Measurement
Average capital's share of output	$\frac{1}{3}$
Average capital-to-output ratio	3.0
Average investment-to-output ratio	0.20
Average consumption-to-output ratio	0.80

(10 points)

5. Assume now that the number of hours worked h_t is not decided by the individuals, but set equal to $\bar{h}$ by government regulation and could in principle vary from period to period. Write up the new recursive problem. What are now the (endogenous / exogenous) state variable(s) and what are the control variable(s)? (4 points)

6. The individuals have thought that the numbers of hours worked would remain constant ad infinity, $\bar{h} = \frac{1}{3}$, and have made their consumption/investment decisions accordingly.

   However, the government considers cutting the mandatory work-week by 20%; $\tilde{h} = 0.8 \cdot \bar{h}$; *without any prior public announcement.* You have been asked to evaluate the dynamic effects of the potential reform.

   In order to do this you will numerically solve the recursive problem. Then you will study the dynamics of the endogenous variables of the economy following a change of h from $\bar{h}$ to $\tilde{h}$.

   Carefully describe how you would numerically solve for the recursive equilibrium and then use this solution to simulate the economy's response to the change in hours worked. (15 points)

3 : Overlapping Generation Model (40%)

Consider an overlapping generations economy where agents live for two period for sure. The economy starts at time 1. Denote the generation born at time t as cohort t. Each individual has one unit of time to work when young and leisure is valued. More specifically, assume utility of an agent born at t is

$$U_t = \log(c_{1t}) + \psi \log(1 - h_t) + \log(c_{2t+1})$$

where h_t is the labor supply when young. The individual does not work when old.
Population L_t grows at a rate n so that $L_t = (1 + n) L_{t-1}$.
Assume also there is an initial old cohort at time 1 with size $L_0 = 1$.The initial old is endowed with capital stock K_0, which is given exogenously.
A representative firm hire labor and capital in a competitive market and pay wage rate w_t and interest rate r_t. The production function is

$$Y_t = K_t^\alpha H_t^{1-\alpha}$$

where $\alpha \in (0, 1)$, $H_t = L_t h_t$ is the aggregate labor supply at time t. Finally, capital stock does not depreciate after one period so that

$$K_{t+1} - K_t + C_t = Y_t$$

where C_t is aggregate consumption.

1. Write down the problem for a young agent of cohort t $(t \geq 1)$. Define a competitive equilibrium for this economy. (7 points)

2. Given factor prices, solve for the consumption, saving and hours worked for a cohort t agent in this economy. (7 points)

3. Suppose at some time τ, the government announces that starting from period $\tau+1$, a Pay-As-You-Go (or "PAYG") social security system will be introduced into this economy for ever. The payroll tax is a lump-sum tax T and shall be imposed on each worker starting from time $\tau + 1$. All agents alive at period τ learns this news at time τ. Solve for the consumption allocations and hours worked for an agent born at time τ and $\tau + 1$ as functions of factor prices and T. Using your results, explain how an introduction of PAYG social security systems changes the hours worked per individual, h, for cohorts born at both time τ and $\tau + 1$. (9 points)

4. Assume, for simplicity, now that leisure is not valued by agents so that $h_t = 1$ for all $t \geq 1$. Accordingly, the life time utility becomes

$$U_t = \log(c_{1t}) + \log(c_{2t+1})$$

Suppose the government announces at time τ that a fully-funded social security system will be introduced from time $\tau + 1$ on forever. Again, the payroll tax is a lump-sum tax T. This implies that each agent born at any future time $t \geq \tau + 1$ will receive a pension benefit of a size $T(1 + r_{t+1})$ when she becomes old. Solve for the consumption and savings allocations of an agent born at time $\tau + 1$ as functions of factor prices and T. (7 points)

5. Continue with Part 4 above. Use the capital market clearing condition to derive the law of motion for capital per worker for all future periods $t \geq \tau + 1$, after the fully-funded system is introduced. Solve for the steady-state level of aggregate capital stock per worker in the economy with the fully-funded system. How does the introduction of a fully-funded social security system affect the aggregate capital stock in the steady state? Explain why. (10 points)