# *UNIVERSITY OF OSLO*
# *DEPARTMENT OF ECONOMICS*

Exam: **ECON4310 – Intertemporal macroeconomics**

Date of exam: Monday, December 7, 2009 **Grades are given: January 6, 2010**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 3 pages (incl. cover sheet)

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Question 1 has weight 2/3, question 2 1/3.

# 1

In this question we shall be looking at a discrete-time version of Ramsey's growth model. The social planner maximizes:

$$U = \sum_{t=0}^{\infty} \beta^t u(\bar{c}_t) \tag{1}$$

where $\bar{c}_t$ is consumption per capita and $0 < \beta < 1$ is a subjective discount factor. The period utility function is

$$u(c) = c^{1-\theta}/(1-\theta) \tag{2}$$

where $\theta > 1$.

The aggregate production function is

$$Y_t = F(K_t, A_t L_t) \tag{3}$$

where $Y_t$ is output, $K_t$ is the capital stock, $L_t$ the labor force and $A_t$ a labor-augmenting productivity factor. $F$ is assumed to be homogeneous of degree one. (If you need additional assumptions about $F$ to answer the questions below, you should state them as part of your answer). $L_t$ is exogenous and grows with a rate $n$ per period ($L_{t+1} = (1+n)L_t$). Similarly, $A_t$ is exogenous and grows with rate $g$. The capital stock grows according to

$$K_{t+1} = K_t - \delta K_t + Y_t - C_t \tag{4}$$

where $\delta$ is the depreciation rate, and $C_t = L_t \bar{c}_t$ is aggregate consumption. For every $t$ it is required that $K_t \geq 0$ and $C_t \geq 0$.

1. Define $k_t = K_t/A_t L_T$ and $c_t = \bar{c}_t/A_t$. Explain in words what these variables measure. Show that the social planner's problem can be expressed as

   $$\max U = \sum_{t=0}^{\infty} \beta^t \frac{(A_t c_t)^{1-\theta}}{1-\theta} \tag{5}$$

   given that

   $$(1+n)(1+g)k_{t+1} = (1-\delta)k_t + f(k_t) - c_t \qquad t = 0, 1, 2, \ldots \tag{6}$$

   and given $k_0$.

2. Use the Bellman equation to show that the first-order condition for an internal optimum can be written

   $$c_t^{-\theta} = \beta c_{t+1}^{-\theta} \frac{1 + f'(k_{t+1}) - \delta}{(1+g)^{\theta}(1+n)} \tag{7}$$

   Interpret this condition. What does it say about the growth rate of consumption?

3. Explain what is meant by a balanced growth path (a steady state). Does a balanced growth path exist in the present model? What determines the values of $k$ and $c$ along a balanced growth path? What role does intertemporal substitution play for these steady-state values?

4. Compare the values of $k$ and $c$ along the balanced growth paths for two economies that have different productivity growth $g$.

5. What is meant by stability of the steady state? Is the steady state in the present model stable? How are the initial levels of $k$ and $c$ determined? Illustrate with a graph.

6. Suppose the economy is initially on a balanced growth path. Discuss with the help of a graph the effect of an increase in $g$ on the whole time path for $k_t$.

7. Assume that $F(K, AL)$ has a Cobb-Douglas structure and that $k$ only can take on 10 different values. $k1, \ldots, k10$. What is the structure of the value function you used in 2) in this case? Explain how you would calculate the value function using Matlab.

8. How would you use the result in 7) to study the convergence to steady state from some initial capital level $k$?

## 2

Discuss the treatment of the labor market in real business cycle theories and how these theories explain fluctuations in employment.