ECON4310 Fall 2010 For the examiners

January 5, 2011

Question 1 counts 3/4, question 2 1/4.

1 Overlapping generations

1. Budget constraint:
$$c_{y,t} + \frac{1}{1+r_{t+1}} c_{o,t+1} = w_t \tag{1}$$
First-order condition:
$$c_{y,t}^{-\theta} = c_{o,t+1}^{-\theta} \beta (1 + r_{t+1}) \tag{2}$$

2. Savings rate
$$s_t = (w_t - c_{y,t})/w_t$$
From first-order condition
$$c_{o,t+1} = c_{y,t}[\beta(1 + r_{t+1})]^{1/\theta}$$
Insert in budget equation and solve:
$$c_{o,t+1} = \frac{w_t}{1 + \beta^\sigma (1 + r_{t+1})^{\sigma - 1}}$$
where $\sigma = 1/\theta$ Hence,
$$s_t = \frac{\beta^\sigma (1 + r_{t+1})^{\sigma-1}}{1 + \beta^\sigma (1 + r_{t+1})^{\sigma-1}} = \frac{1}{\beta^{-\sigma}(1 + r_{t+1})^{1-\sigma} + 1} \tag{3}$$
Since $\theta > 1$, $\sigma < 1$ and r has negative effect on saving. (no need to compute derivative).

   $1/\theta$ is substitution elasticity and determines the strength of the substitution effect.

   Give credit also for more informal discussions on this point.

3. Explanation starts with that the capital stock in period $t + 1$ is equal to the savings of the young in period t, $(1 + n)N_t k_t = N_t s_t w_t$

   Producer behavior implies $r_{t+1} = f'(k_{t+1})$ and $w_t = f(k_t) - f'(k_t)k_t$.

4. Look for general explanations of the concepts. Stationary values of k are values that solve
$$k = \frac{s(f'(k))}{1+n}[f(k) - f'(k)k] \tag{4}$$
A particular stationary point $\bar{k}$ is stable if $-1 < dk_{t+1}/dk_t < 1$ when calculated in $\bar{k}$. A brief answer along these lines should be sufficient. Some may have calculated the derivate involved by implicit derivation of the differential equation, which yields
$$\frac{dk_{t+1}}{dk_t} = \frac{-s(\bar{r})f''(\bar{k})\bar{k}}{1+n-s'(\bar{r})f''(\bar{k})\bar{w}}$$
but this is not expected.

5. Standard graph with k_t and k_{t+1} on the axis, a 45-degree line and a curve representing the difference equation. This curve starts from the origin and is upward sloping and concave with one intersection point with 45-degree line.

6. $$\tau_t = g + (r_t - n)b$$

7. $$k_{t+1} + b = \frac{s(f'(k_{t+1}))}{1+n}[f(k_t) - f'(k_t)k_t - (g + (r_t - n)b)] \tag{5}$$

8. Direct crowding out effect (b on lhs. above). Indirect effect through taxes, ambiguous, probably minor relative to direct effect. Curve in graph shifts down. Lower k and higher r in the stable equilibrium.

9. Hardly. (Perhaps "every" should have been "any"). If the curve in the graph shifts down sufficiently there is no intersection. The general intuition is that consumers are saving for their own retirement only, and that there are limits too how much consumers will save for this purpose. Hence there are limits to how much government debt consumers will absorb, especially since higher interest rates do not make them willing to save more. High savings require a high capital stock which gives higher wages and lower interest rates. However, the room for government debt is determined by the difference between the savings of the young and the capital stock. That the marginal productivity declines towards zero ensures that the room for government debt has a maximum.

   The question is rather peripheral. Do not expect too much.

   [More formal arguments (not expected) are best made by example. The easiest to look at is the case where $\sigma = 1$, $\beta = 0$, n =0, $g = 0$ and $f(k) = k^\alpha$. The savings rate is then constant and the stationarity condition can be reduced to
$$k + b = s[(1-\alpha)k^\alpha - \alpha k^{\alpha-1}b]$$

which can be solved for b:

$$b = \frac{s(1-\alpha)k^{\alpha} - k}{1 + s\alpha k^{\alpha-1}}$$

One can then show that the right hand side always has a maximum, which becomes an upper limit on b. In fact, since the denominator is always greater than one, the expression is bounded above by $s(1-\alpha)k^{\alpha}-k$, which has a unique maximum value. This can be interpreted as the potential for saving in excess of what is needed to keep the capital stock intact.]

2 Ricardian equivalence

The time constraint plus the 1/4 weight means that formal analysis cannot be expected. The lectures and texts have discussed fiscal policy in Ramsey and Diamond models. Romer Ch 11.2 has a more focused discussion. Look for relevant points; do not expect an exhaustive discussion.