## UNIVERSITY OF OSLO DEPARTMENT OF ECONOMICS

Exam: ECON4310 - Macroeconomic theory
Date of exam: Thursday, December 2, $2010 \quad$ Grades will be given: December 21, -10
Time for exam: 2:30 p.m. - 5:30 p.m.
The problem set covers 3 pages (incl. cover sheet)
Resources allowed:

- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Please do not forget the periodic course evaluation for ECON4310, which you will find on the website for the course. The deadline is December 19!

Question 1 counts $3 / 4$, question $21 / 4$.

## 1 Overlapping generations

Consumers live for two periods. When young they supply one unit of labor, when old they retire. They start life without any financial assets and leave no bequests. Consumers who are young in period $t$ earn the wage $w_{t}$. They consume $c_{y, t}$ when young and $c_{o, t+1}$ when old. The consumers have utility functions

$$
\begin{equation*}
U=u\left(c_{y, t}\right)+\beta u\left(c_{o, t+1}\right), \quad 0<\beta<1 \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
u(c)=c^{(1-\theta)} /(1-\theta), \quad \theta>1 \tag{2}
\end{equation*}
$$

There are no restrictions on borrowing and lending, except that the consumers must repay all their debts before they die. The real interest rate from period $t$ to $t+1$ is $r_{t+1}$.

1. Write down the consumers' budget constraint and derive the first-order conditions for utility maximization.
2. Derive an expression for the savings rate, $s_{t}$, of a young consumer. What are the forces that determine the amount of savings? How does the savings rate depend on the real interest rate? What role does the parameter $\theta$ play?

We now go on to look at the aggregate economy. There is no productivity growth. The young generation at $t$ consists of $N_{t}$ individuals. The population grows with a constant rate $n$. Output per worker, $y_{t}$ is given by the production function

$$
\begin{equation*}
y_{t}=f\left(k_{t}\right) \tag{3}
\end{equation*}
$$

where $k_{t}$ is the capital stock per worker and $f$ has standard properties. There is no depreciation.
3. Show that the time path of $k_{t}$ can be described by the equation

$$
\begin{equation*}
k_{t+1}=\frac{s\left(f^{\prime}\left(k_{t+1}\right)\right)}{1+n}\left[f\left(k_{t}\right)-f^{\prime}\left(k_{t}\right) k_{t}\right] \tag{4}
\end{equation*}
$$

where the function $s$ is the relationship you were asked for in question 2 between the savings rate of a young consumer and the real interest rate $\left(s_{t}=s\left(r_{t+1}\right)\right)$. Interpret the equation.
4. Explain what is meant by a steady state and a stationary equilibrium. How are the stationary value(s) of capital per worker determined? Explain also what it means that a certain stationary equilibrium is stable. What is the condition for (local) stability of a stationary point in (4)? (Brief answers will do).
5. Draw a graph that illustrates how $k_{t}$ evolves over time from an arbitrary starting point $k_{0}$ when there is only one stationary point with a strictly positive capital stock and this is stable.
6. We now introduce government debt in the model. Government consumption per young consumer is $g$, government debt $b$. The government collects a lump-sum tax per young consumer, $\tau_{t}$. This is set in order to keep the debt per young individual constant. What is the required level of the tax?
7. How will you modify equation (4) in order to include the effect of the government debt on the evolution of the capital stock?
8. Assume again that there is only one stable steady state with a strictly positive level of capital. What is the effect of increased government debt on the level of the capital stock per worker in the steady state? What does this mean for the level of the interest rate? (A good intuitive explanation is sufficient).
9. Does a steady state exist for every level of government debt?

## 2 Ricardian equivalence

Explain what is meant by Ricardian equivalence and under what conditions it may apply. Discuss the relevance of the Ricardian equivalence proposition for decisions about tax levels in different countries today.

