ECON4310 2011 For the examiners

Remember that this is a three-hour exam. Do not expect too much detail and do not punish those who run out of time too hard. If grades on A and B differ, make an overall judgment.

A

Sources from the reading list: Roemer, chapter 2, part A and Williamson, chapter 3

1. Define $y_t = Y_t/N_t$ and $k_t = K_t/N_t$. Since (3) is homogeneous of degree one,

$$y_t = \frac{Y_t}{N_t} = F\left(\frac{K_t}{N_t}, \frac{N_t}{N_t}\right) = F(k_t, 1)$$

Define the production function in intensive form as $f(k_t) = F(k_t, 1)$. Then

$$y_t = f(k_t) \tag{5}$$

Divide by through (4) with N_t to get

$$\frac{K_{t+1}}{N_t} = \frac{K_{t+1}N_{t+1}}{N_t N_{t+1}} = \frac{K_t}{N_t}(1-\delta) + \frac{Y_t}{N_t} - c_t$$

or, taking account of (5) and that $N_{t+1} = (1+n)N_t$

$$k_{t+1}(1+n) = k_t(1-\delta) + f(k_t) - c_t, \qquad t = 0, 1, 2, \ldots \tag{6}$$

2. First order conditions can be derived by straightforward maximization of (1) with (6) as constraints or by using the Bellman equations. Maximization is with respect to $c_0, c_1, c_2, \ldots$ and $k_1, k_2, k_e, \ldots$. The simplest way of handling the constraints is to use them to substitute for c_t in the utility function (1) and then maximize with respect to the ks. The alternative is to use Lagrange multipliers, one for each t. If the Bellman method is used, k_t is the state variable. As long as the derivation is correct, the method used should not influence the grading.

In the sequel the case where the utility function is (1) will be denoted "A", while the case corresponding to (2) will be case "B". The firsts order conditions for an internal maximum can be written

$$A: \qquad u'(c_t) = u'(c_{t+1})\frac{1+f'(k_{t+1})-\delta}{1+\rho} \tag{7}$$

$$B: \qquad u'(c_t) = u'(c_{t+1})\frac{1+f'(k_{t+1})-\delta}{(1+\rho)(1+n)} \tag{8}$$

The left hand side is the gain from a marginal increase in consumption in period t. The right hand side is the gain from transferring the same amount of goods to period $t+1$ to use as capital and then consuming the gross returns from the investment. When the first order conditions says that these should be equal, it means that in optimum it should not be possible to gain in utility by transferring resources between two consecutive periods. The increase in per capita consumption in period $t+1$ that is made possible by a marginal transfer of resources from period t is $[1-\delta+f'(k_{t+1})]/(1+n)$. The marginal utility that applies to this increase in consumption is $u'(c_{t+1})$. This is discounted by the factor $1/(1+\rho)$ because the social planner is impatient and values consumption in period t higher than in period $t+1$. Case A differs from case B in that the social planner values per capita consumption higher when there are more consumers to take advantage. Population growth then has the opposite effect of discounting. The higher weight on larger generations in case A, neutralizes the effect of that resources saved in t will be divided on more consumers in period $t+1$. This is the reason that the factor $1+n$, which is visible in case B, disappear in case A. The main difference between the two first-order conditions is then that, everything else equal, the right hand side is lower in case B.

Assuming that u is concave

$$c_{t+1} > c_t \Leftrightarrow u'(c_t) > u'(c_{t+1})$$

From (7) and (8) we then get conditions for consumption to be growing.

$$A \qquad f'(k_{t+1}) - \delta > \rho$$

$$B \qquad f'(k_{t+1}) - \delta > \rho + n + \rho n$$

In case A the marginal product of capital net of depreciation should be high enough to overcome the impatience of the social planner. In case B population growth makes the hurdle higher. Social planner B requires a higher return to be willing to increase per capita consumption.

3. A steady state is when the per capita variables k_t, c_t, y_t are constant from period to period while the aggregates such as K_t and Y_t grow with the same rate as the population. We can determine the steady state by setting $k_{t+1} = k_t = k$ $c_{t+1} = c_t =$ in the first-order condition and the constraint. From the former we get:

$$A \qquad f'(k) = \delta + \rho$$

$$B \qquad f'(k) = \delta + \rho + n + \rho n$$

These equations alone determine k. Concavity of the production function means that k is higher in case A than in case B. The constraint ( ) then gives us steady state consumption as

$$c = f(k) - (\delta + n)k$$

Steady state consumption is increasing in k, since $f'(k) > \delta + n$ in both cases. This means that c is highest in case A.

4. A phase diagram is attached. Arrows that show the directions of movement should be added. Since the two cases look qualitatively the same, one phase diagram is enough. (In the course we have drawn the curve for c constant vertical even if this is not strictly correct. This was done to make the graphs look exactly like the continuous time graphs in Romer. The qualitative aspects of the analysis are not affected).

5. The curve representing constant capital stock shifts down. The curve representing a constant consumption stays put in case A and shifts left in case B. In case A the steady state level of the capital stock does not change while consumption is reduced immediately to the new steady state level. There are no further dynamics. In case B the new steady state has a less capital and lower consumption. Consumption will first jump up to the saddle path and then consumption and capital will decline gradually towards the new steady state.

B

Sources: From the reading list: Sørensen and Whitta-Jacobsen, chapter 14, on question 2 also first sections of Romer's chapter on investment. Questions 1 and 3 can also be answered from standard undergraduate theory of markets.

1. Main points:

   - In the short run the supply curve is vertical.
   - House prices are determined by the intersection between the vertical supply curve and a downward sloping demand curve
   - Housing investment (the rate of construction of new houses) is determined by the intersection between the house price and an upward sloping supply curve for new houses
   - Over time new construction will move the increase the supply of housing and reduce house prices (everything else equal)

   Extra credit should be given for more elaborate descriptions of what is behind the demand and supply curves. Award insights independently of whether they are formalized or not.

2. Tobin's q-theory basically says that investment in new capital is determined by the ratio between market price of existing, installed capital and the cost of acquiring new capital goods. Behind this is an assumption that installation costs for new capital are increasing with the rate of investment. This means that the capital stock will be changed only gradually over time and that the implicit supply curve for new installed capital is increasing

in the price of capital. Hence, the mechanisms at play in the two theories are very similar. The two theories differ in the detailed description of the mechanisms that makes the supply curve for new capital increasing. While house prices are directly observable, the current value of the installed capital in firms is not. In Tobin's q-theory it is assumed that the latter can be inferred from share prices.

3.
   - Demand curve for existing housing shifts left.
   - House prices go down
   - Investment in new housing goes down.
   - Over time house prices will increase again.