Question 1

a)   Budget constraint:

$$c_1 + w_1\ell_1 + \frac{c_2 + w_2\ell_2}{1+r} = w_1T + \frac{w_2T}{1+r}(= F)$$

By Lagrange's method first order conditions are (assuming interior solution)

$$\frac{1}{c_1} = \lambda, \ \frac{\psi}{\ell_1}, = \lambda w_1, \ \frac{\beta}{c_2} = \lambda\frac{1}{1+r}, \ \frac{\beta\psi}{\ell_2} = \lambda\frac{w_2}{1+r},$$

b)   Solve the five equations. One way is to insert for c_1, ℓ_1... from the first order condition in the budget constraint and then use the latter to determine λ. Solution:

$$
\begin{aligned}
c_1 &= (1/\mu)[w_1 + w_2(1+r)^{-1}]T \\
c_2 &= \beta/\mu[(1+r)w_1 + w_2]T \\
\ell_1 &= (\psi/\mu)[1 + \frac{w_2}{w_1}(1+r)^{-1}]T \\
\ell_2 &= (\beta\psi/\mu)[\frac{w_1}{w_2}(1+r) + 1]T
\end{aligned}
$$

where $\mu = 1 + \psi + \beta + \beta\psi$.

c) $w_1 = z_1$, $w_2 = z_2$

d)   Doubling of z_2

1. Labor supply up in period 2, down in period 1. Intertemporal substitution. Work when wage is high, leisure when wage is low.

2. Consumption up in both periods. Consumption smoothing.

e)   Here there is room for more or less comprehensive answers. We expect most students to say something about the interest rate going up and that the quantitative effects are dampened.

f)   Wage subsidy. If markets are competitive, wages paid by the firm do not change. Labor supply will shift from the first to the second period. Consumption will increase in both periods. (Or stay constant if subsidy is financed by lump-sum tax).

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Question 2

1. Interpretation straightforward. Society's resource constraint is sum of resource constraints for the two sectors..

2. Straightforward. There is no preference for any particular mathematical method. (Bellman Equation and value function was not on the required curriculum this year).

3. Optimal consumption

   (a) Insertion in the first order condition yields

   $$c_t = c_0(1+g)^t$$

   (Discounting and diminishing marginal utility should tilt consumption forward relative to wages, but this is counterbalanced the interest rate by assumption). Inserting the last expression in the resource constraint yields:

   $$\sum_{t=0}^{\infty} \frac{c_0(1+g)^t}{(1+r)^t} = (1+r)A_0 + \sum_{t=0}^{\infty} \frac{w_0(1+g)^t}{(1+r)^t}$$

   Solving for c_0 yields:

   $$c_0 = (r-g)A_0 + w_0$$

   Insert in c_t

   $$c_t = (r-g)A_0(1+g)^t + w_t$$

   (b) Optimal fiscal rule: $T_t = -(r-g)A_t$

   (c) Wealth: Fiscal rule inserted in government budget equation yields

   $$A_{t+1} = (1+g)A_t$$

   or

   $$A_t = A_0(1+g)^t$$

   which remains positive forever.

4. Fall in interest rate. In questions (a) and (b) the intention was that students should look at cases where $(1+\rho) = (1+r)/(1+g)$ continues to hold. Below a solution based on this interpretation is suggested. Interpretations $(1+\rho) \neq (1+r)/(1+g)$ after the change in parameters should also be accepted. Their efforts should be recognized according to the difficulty of the task. Good answers based on the second interpretation, may be along the same line as the suggested answer to (c).

(a) If $(1 + \rho) = (1 + r)/(1 + g)$ holds, the solution formulas in 3 are still valid. Consumption starts at (approximately) the same level. Lower growth rate in both consumption and assets. [If $(1 + r) = (1 + \rho)(1 + g)$ is to hold, we must have $\Delta r = (1 + \rho)\Delta g \approx \Delta g$ where $\approx$ holds when the model divides time in short periods. Given the text, no change in initial consumption should also be treated as a correct answer].

(b) Solution formulas in 3 are still valid. Consumption path shifts down, but growth rate remains the same

(c) Solution from 3 no longer valid. Back to the Euler equation. Consumption growth rate is now

$$\frac{1 + r}{1 + \rho} < (1 + g)$$

5. Lower ρ means the government wants to transfer less to the first generations, which means consumption starts lower, and the growth rate is higher. In order to support consumption growth above wage growth, assets must grow faster than g Consumption grows less than wages. Given the resource constraint, this means that consumption has to start higher than wages and end up below. This means that after some time the government will become a borrower and hit the borrowing constraint. After that the consumption growth rate will increase.

Questions 4 and 5 are rather difficult. Hence, shortcomings here should have low weight.