

***UNIVERSITY OF OSLO***  
***DEPARTMENT OF ECONOMICS***

Exam: **ECON4310 – Macroeconomic Theory**

Date of exam: Wednesday, December 4, 2013      **Grades are given: January 3, 2014**

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 4 pages (incl. cover sheet)

Resources allowed:

- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

# 1 Real Business Cycles (Weight 50 per cent)

Assume a consumer living for two periods in a small open economy with the utility function

$$U = \ln(c_1) + \psi \ln(l_1) + \beta[\ln(c_2) + \psi \ln(l_2)]$$

where  $c$  is consumption and  $l$  is leisure. Total time available per period amounts to  $T$ . The real wage equals  $w_1$  in period 1 and  $w_2$  in period 2. The consumer has no capital income and starts life without any assets. The consumer may transfer income between periods at the exogenous world interest rate  $r$ .

- a) Set up the optimization problem of this consumer and derive the first-order conditions. Note that savings are endogenous.
- b) Derive the demand for leisure in each period as a function of only exogenous parameters.
- c) Assume there is no capital in the economy, and firms produce with the production function  $Y_t = z_t N_t$  for  $t \in \{1, 2\}$ , where  $N_t$  is labor input in period  $t$   $z$  is productivity. What are the competitive equilibrium wage rates in the two periods?
- d) Assume that the consumer anticipates in period 1 a boom in period 2, this means  $z_2$  doubles to  $2z_2$ . How do wages change in the two periods? How do labor supply and consumption change? Explain the intuition of your findings.
- e) Consider the same experiment as in (d) in a closed economy where production uses both capital and labor. Think of a general equilibrium real business cycle model so that the interest rate is endogenously determined within the country. Explain intuitively what changes relative to the case of an open economy without capital.
- f) Consider again the consumer problem without capital and with exogenous interest rate (small open economy). Assume that in the second period the government decides to pay a subsidy to labor income of 100 % , that is for every NOK you earn the government pays an extra NOK, so that the wage for consumers double. How do the wages paid by the firm in the two periods change? How does labor supply and consumption change. Explain the intuition of your findings. Hint: this does not involve any additional math.

## 2 Optimal fiscal policy(Weight 50 per cent)

Consider an economy where each generation lives for one period. The generation born in period  $t$  has a wage  $w_t$ , and this wage grows at rate  $g$  over time;

$$w_t = w_0 (1 + g)^t.$$

All generations have preferences over consumption, given by  $u(c_t) = \log(c_t)$ . There is a long-lived government which cares about all generations, attaching weight  $\beta^t$  on the generation born in period  $t$ . The government can redistribute between generations by issuing (positive or negative) lump sum taxes  $T_t$  on each generation. Thus, the budget constraint for each generation is given by  $c_t = w_t - T_t$ .

The government can borrow and lend at the world market at the constant world interest rate,  $r$  (assumed to be larger than  $g$ ). Thus, the government's budget constraint is

$$A_{t+1} = (1 + r) A_t + T_t,$$

where  $A_t$  denotes the financial wealth that the government carries over from period  $t - 1$  to period  $t$ . The government also faces a borrowing constraint  $A_t \geq -\underline{A}$ , where  $\underline{A} > 0$  is the maximum amount that can be borrowed. This prevents Ponzi-schemes. The government's optimization problem is then

$$\max \sum_{t=0}^{\infty} \beta^t \ln(c_t),$$

subject to a resource constraint and the borrowing constraint.

1. The resource constraint for the planner can be written as

$$\sum_{t=0}^{\infty} \frac{c_t}{(1 + r)^t} \leq (1 + r) A_0 + \sum_{t=0}^{\infty} \frac{w_0}{(1 + r)^t} (1 + g)^t.$$

Interpret this expression and explain in words why this is the relevant resource constraint.

2. Show that the planner's Euler equation is

$$\frac{1}{c_t} = \beta (1 + r) \frac{1}{c_{t+1}},$$

assuming that the borrowing constraint is not binding.

3. The discount rate  $\rho$  is defined by  $\beta = 1/(1 + \rho)$ . Assume that  $\rho$  satisfies  $(1 + \rho) = (1 + r)/(1 + g)$ . Assume also that  $A_0 > 0$ , maybe because of an oil discovery.

- (a) Derive the consumption path the planner will choose.
  - (b) What is the optimal "fiscal rule" (the government's primary deficit as a function of current wealth) in this case?
  - (c) What happens to the level of government financial wealth,  $A_t$ , in the long run? Will the borrowing constraint ever become effective?
4. After year 2000, the interest rates on risk free government bonds (e.g. US and German debt) have fallen. We are interested in the effects of a permanent fall in  $r$ , starting from an initial situation where  $(1 + \rho) = (1 + r)/(1 + g)$  is satisfied as in question 3.
- (a) Suppose that the fall in  $r$  were caused by an equal and simultaneous fall in the growth rate,  $g$ . How would this have affected the optimal time paths of consumption and government financial wealth?
  - (b) Would the effects be different if instead there was a simultaneous and equal fall in  $r$  and  $\rho$  with the growth rate,  $g$ , constant?
  - (c) Discuss the effects of a reduction in  $r$  if there is no change in  $\rho$  nor  $g$ .
5. Ramsey's normative ideal was to attach the same weight to all generations (so  $\beta = 1$ ,  $\rho = 0$ ). The planner may want to live up to this. What happens to long-run government wealth,  $A_t$ , and initial consumption,  $c_0$ , as  $\rho$  approaches zero (assuming that  $r$  remains unchanged)?