UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam: **ECON4310 – Macroeconomic Theory**

Date of exam: Monday, December 14, 2015

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 13 pages (incl. cover sheet)

Resources allowed:

- No written or printed resources – or calculator - allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences)

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Final Exam II
ECON 4310, Fall 2015

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	**Max**
Exercise A		**60**
Exercise B		**60**
Exercise C		**60**
Σ		**180**

Exercise A:
Short Questions (60 Points)

Answer each of the following short questions on a seperate answer sheet by stating as a first answer True/False and then give a short but instructive explanation. You can write, calculate, or draw to explain your answer. You only get points if you have stated the correct True/False *and* provided a correct explanation to the question. We will not assign negative points for incorrect answers.

Exercise A.1: (10 Points) Finite horizon model of intertemporal consumption

Consider the optimal intertemporal consumption choice of a household in discrete and finite time $t = 0, 1, \dots, T < \infty$. The optimal behavior is characterized by the consumption Euler equation

$$\frac{c_{t+1}}{c_t} = [\beta(1 + r - \delta)]^{1/\theta}, \forall t \leq T,$$

and the private bugdet constraint

$$a_{t+1} + c_t = (1 + r - \delta)a_t + w, \forall t \leq T,$$

taking as given initial and terminal conditions

$$a_0 = 0$$
$$a_{T+1} = 0,$$

where a_t denotes asset holdings, $r - \delta$ is the exogenous interest rate, w the constant and exogenously given wage income, c_t the individual consumption of the household, $\delta \in (0, 1)$ the depreciation rate of physical capital, $\beta \in (0, 1)$ is the subjective discount factor, and $1/\theta$ the intertemporal elasticity of substitution.

Suppose that $\beta(1 + r - \delta) = 1$. Then this household will first accumulate strictly positive assets for a while and then run down assets over the life-cycle. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.2: (10 Points) OLG model, a great famine

Consider the capital accumulation equation of the overlapping generations model with exogenous technology and population growth.

$$k_{t+1} = \frac{(1-\alpha)\beta}{(1+\beta)(1+g)(1+n)} k_t^{\alpha}, \quad k_t \equiv K_t/(A_t L_t),$$

where K_t is the aggregate capital stock, A_t is the state of technology, L_t the size of the young cohort, $\beta \in (0,1)$ the discount factor, $\alpha \in (0,1)$ the capital income share in the economy, and $g \geq 0$ and $n \geq 0$ denote the net growth rate of technology and the cohort size, respectively. The competitive interest rate is given by

$$r_t - \delta = \alpha k_t^{\alpha-1} - \delta.$$

Let the economy be in the stable steady-state with capital per efficiency unit, $k^\star \equiv K_t/(A_t L_t) > 0$. Now assume that in period t_0, unexpectetly, a great famine hits this economy such that the current cohort of young is reduced from L_{t_0} to $L'_{t_0} < L_{t_0}$. In response to this unexpected shock the interest rate will jump down on impact and then increase as the economy converges gradually back to the initial steady-state. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.3: (10 Points) Ramsey model, technology shock

Consider the dynamic equilibrium equations of the Ramsey model with exogenous growth in technology

$$\frac{c_{t+1}}{c_t} = \left[\frac{\beta}{1+g}(1+\alpha k_{t+1}^{\alpha-1}-\delta)\right]^{1/\theta}, \quad c_t \equiv C_t/(A_t L),$$
$$k_{t+1}-k_t = k_t^{\alpha}-c_t-\delta k_t - g k_{t+1}, \quad k_t \equiv K_t/(A_t L),$$

where K_t is the aggregate capital stock, $\alpha k_{t+1}^{\alpha-1}-\delta$ the interest rate, $g \geq 0$ is the constant net growth rate of technology,

$$A_{t+1} = (1+g)A_t,\ A_0 > 0,$$

$L = 1$ the constant size of the population, C_t aggregate consumption, $\alpha \in (0,1)$ the capital income share in the economy, $\delta \in (0,1)$ the depreciation rate of physical capital, $\beta \in (0,1)$ is the subjective discount factor, and $1/\theta$ the intertemporal elasticity of substitution.

Suppose that the economy is in the steady-state with capital stock per efficieny unit, $k^{\star} \equiv K_t/(A_t L)$. Now assume that in period t_0, the level of technology jumps up from A_{t_0} to $A'_{t_0} > A_{t_0}$ due to an unexpected innovation. In response to this temporary one-period shock consumption per efficiency unit, c_t, will jump down on impact. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.4: (10 Points) Ramsey model, Golden Rule capital stock

Consider the capital accumulation equation of the Ramsey model with exogenous technology and population growth

$$(1+g)(1+n)k_{t+1} = k_t^\alpha - c_t + (1-\delta)k_t, \quad k_t \equiv K_t/(A_t L_t),\, c_t \equiv C_t/(A_t L_t),$$

where K_t is the aggregate capital stock, A_t is the state of technology, L_t the size of the population, C_t aggregate consumption, $\alpha \in (0,1)$ the capital income share in the economy, $\delta \in (0,1)$ the depreciation rate of physical capital, $g \geq 0$ denotes the net growth rate of technology, and $n \geq 0$ is the net growth rate of population. The compettitve interest rate is given by

$$r_t - \delta = \alpha k_t^{\alpha-1} - \delta.$$

The Golden Rule capital stock per efficiency unit (the capital stock per efficiency unit that maximizes steady-state consumption per efficiency unit) implies an interest rate of

$$r_{GR} - \delta = (1+g)(1+n) - 1.$$

True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.5: (10 Points) Precautionary savings motive

Consider the simple two-period real business cycle model discussed in the seminar and in the lecture. With an asset supply of zero, $w_1 = \mathrm{E}[w(s_2)]$, and an optimal consumption profile, $c_1 = w_1$, $c_2(s_2) = w(s_2)$, the stochastic consumption Euler equation in this model is given by

$$\beta(1 + r_2) = \frac{u'(c_1)}{\mathrm{E}[u'(c_2(s_2))]} = \frac{u'(\mathrm{E}[w(s_2)])}{\mathrm{E}[u'(w(s_2))]}.$$

The stochastic process for the wage in the second period, $w(s_2)$, takes the form

$$w(s_2) = \begin{cases} w(s_G) = 1 + \sigma/2, & \text{with prob. } 1/2, \\ w(s_B) = 1 - \sigma/2, & \text{with prob. } 1/2, \end{cases}$$

where $\sigma \in [0, 2)$ parametrizes the risk in this economy. Assume that the utility function is of the following form

$$u(c) = 1 - \frac{1}{\alpha} e^{-\alpha c}, \ \alpha > 0.$$

This utility function, $u(c)$, implies that there is a precautionary savings motive in this economy. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.6: (10 Points) Precautionary savings and labor supply

Consider the simple two-period real business cycle model discussed in the seminar and in the lecture. The stochastic consumption Euler equation reads

$$\beta(1+r_2) = \frac{u'(c_1)}{\mathrm{E}[u'(c_2(s_2))]}.$$

Let the asset supply be fixed to zero such that the optimal consumption is $c_1 = w_1$ and $c_2 = w(s_2)$, and let the utility function $u(c)$ be such that there is a precautionary savings motive in this economy (you don't need to check this).

The stochastic process for the wage in the second period, $w(s_2)$, takes the form

$$w(s_2) = \begin{cases} w(s_G) = 1 + \sigma/2, & \text{with prob. } 1/2, \\ w(s_B) = 1 - \sigma/2, & \text{with prob. } 1/2, \end{cases}$$

where $\sigma \in [0, 2)$ parametrizes the risk in this economy. Moreover, the wage in period 1 corresponds to the expected wage in period 2, $w_1 = E[w(s_2)]$.

The equilibrium interest rate $1 + r_2$ will be equal to $1/\beta$ in an economy without risk, $\sigma = 0$, and must be strictly lower than $1/\beta$ in an economy with strictly positive risk, $\sigma = 1$. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise B:
Long Question (60 Points)

Consumption response to income shocks

Consider a household decision problem under uncertainty, when preferences are linear-quadratic:

$$u(c_t) = c_t - \frac{b}{2}c_t^2, \quad b \geq 0.$$

The household lives from period 0 to period $T < +\infty$ and discounts the future with factor $\beta = 1$. Assume $1 + r = 1/\beta = 1$, that is $r = 0$. The lifetime budget constraint of the household viewed from period 0 reads

$$(1+r)\,a_0 + \sum_{t=0}^{T} \frac{I_t}{(1+r)^t} = \sum_{t=0}^{T} \frac{c_t}{(1+r)^t}, \tag{1}$$

and viewed from period 1

$$(1+r)\,a_1 + \sum_{t=1}^{T} \frac{I_t}{(1+r)^t} = \sum_{t=1}^{T} \frac{c_t}{(1+r)^t}, \tag{2}$$

where I_t is income in period t which is uncertain in any previous period $s \leq t$ and only learned in period t. Expected income (or, consumption) in period t viewed from period $s \leq t$ equals $E_s[I_t]$ (or, $E_s[c_t]$), and the Euler equation is given by

$$u'(c_t) = \beta(1+r)E_t\left[u'(c_{t+1})\right], \quad \forall t \leq T-1. \tag{3}$$

(a) (10 Points) Under the given assumptions, (i) show that the Euler equation can be written as

$$c_t = E_t[c_{t+1}], \quad \forall t \leq T-1, \tag{4}$$

and, (ii) give an interpretation (just one sentence!) of Equation (4).

(b) (10 Points) Note that Equation (4) together with the law of iterated expectations implies that

$$E_s[c_t] = c_s, \quad \forall s \leq t.$$

Now, assume that the household expects to not receive any subsidy such that income I_t is equal to wage income w_t in any period t. Take expectations on the lifetime budget constraints and show that consumption in period 0 and 1 are given by

$$c_0 = (T+1)^{-1}\left(a_0 + \sum_{t=0}^{T} E_0[I_t]\right), \quad E_0[I_t] = E_0[w_t],$$

and

$$c_1 = T^{-1}\left(a_1 + \sum_{t=1}^{T} E_1[I_t]\right), \qquad E_1[I_t] = E_1[w_t],$$

respectively.

(c) (10 Points) Now assume that the household knows (already in period 0) that from period 1 to period T the government pays a subsidy S, so that income is $I_t = w_t + S$ from period 1 onwards. Derive again consumption in period 0 and 1.

(d) (10 Points) Assume now that in period 0 the household expects to never receive a subsidy. In period 1 the household is surprised since the government now pays a subsidy S from period 1 to period T, so that income $I_t = w_t + S$ from period 1 to T. Derive consumption in period 0 and 1.

(e) (10 Points) Assume now that in period 0 the household expects to never receive a subsidy. In period 1 the household is surprised since the government now pays a subsidy S but only in period 1. From period 2 to period T no subsidies are paid. Derive consumption in period 0 and 1.

(f) (10 Points) Explain your results and the differences in consumption for the different scenarios in parts (c), (d), and (e), relative to part (b).

Exercise C:
Long Question (60 Points)

Labor Supply

Consider a representative consumer living for two periods, denoted by $t \in \{1, 2\}$, in a small open economy. The consumer has preferences over consumption, c_t, and the hours of labor supplied, h_t, of the following form

$$U = \log(c_1) - \frac{\phi}{2}(h_1)^2 + \beta \left[\log(c_2) - \frac{\phi}{2}(h_2)^2 \right],$$

where log denotes as always the natural logarithm. The real wage is w_1 in period 1 and w_2 in period 2. The consumer has no capital income in the first period as she starts life without any assets, but the consumer may transfer income between periods (savings) at the exogenous world interest rate r.

(a) (15 Points) Set up the optimization problem of this consumer and derive the optimality conditions. Note that consumption, labor supply, and savings are the choice variables of this optimization problem. (hint: you can do the optimization subject to two period-by-period budget constraints or subject to a single net present value budget constraint.)

(b) (10 Points) Derive the optimal labor supply and consumption in both periods.

- **Hint 1:** combine first the intratemporal optimality conditions and the Euler equation to derive h_2 and c_1 as a function of h_1 and paramters only. Then use the lifetime budget constraint to derive the optimal labor supply in period 1, h_1. From here, you can compute the remaining equilibrium variables.
- **Hint 2:** if you were not able to solve part (a), you can assume that optimal consumption and labor supply is characterized through the intratemporal optimality condition

  $$\phi h_t = c_t^{-1} w_t,$$

  the intertemporal optimality condition (Euler equation)

  $$\frac{c_2}{c_1} = \beta(1 + r),$$

  and the lifetime budget constraint

  $$c_1 + \frac{c_2}{1 + r} = h_1 w_1 + \frac{h_2 w_2}{1 + r}.$$

  and you will be able to solve the following parts of this exercise.

(c) (5 Points) Suppose now that the considered economy was closed, so that the interest rate is endogenously determined within the country and assume that there is no capital and no bond supply in the economy (so, **the equilibrium savings must be zero**). The representative firm produces with the production function

$$Y_t = A_t H_t,$$

where H_t is the firm's labor demand in period t and input factor markets are competitive. What is the equilibrium wage rate in both periods, $t \in \{1, 2\}$?

(d) (10 Points) Still, consider the closed economy described in part (c) where equilibrium savings must be zero. Furthermore, assume that the consumer anticipates in period $t = 1$ a boom in the second period $t = 2$, this means A_2 increases to $A_2' = 2A_2$. How do the optimal labor supply and consumption in both periods change compared to the scenario where the productivity was still A_2? How does the wage in the two periods change? Explain the intuition of your findings (hint: you will be able so solve this exercise even if you struggled before. Work with the intratemporal optimality condition stated in part (b) and the period-by-period constraints

$$\begin{aligned} c_1 &= h_1 w_1 - s \\ c_2 &= h_2 w_2 + (1+r)s, \end{aligned}$$

and anticipate the equilibrium savings behavior, s.)

(e) (10 Points) Consider the same economy and experiment as before that is a boom in the second period $t = 2$, this means A_2 increases to $A_2' = 2A_2$. But now assume that the economy is small open such that the consumer can save at the fixed world interest r (like in the small open economy considered in parts (a) and (b)). How does the optimal labor supply in both periods and optimal consumption in the first period change? Explain intuitively what changes relative to the closed economy case discussed in part (d). (**hint:** if you did not derive the optimal labor supply in part (b), then you can assume that they are given by

$$h_1 = \left[\frac{1+\beta}{\phi}\left(1 + \beta^{-1}\left(\frac{w_2}{w_1(1+r)}\right)^2\right)^{-1}\right]^{1/2} \tag{5}$$

$$h_2 = \left[\frac{1+\beta}{\phi}\left(\beta^2\left(\frac{w_2}{w_1(1+r)}\right)^{-2} + \beta\right)^{-1}\right]^{1/2}, \tag{6}$$

and you will still be able to answer this question.)

(f) (10 Points) Consider again the closed economy setup where the asset supply is zero and the interest rate endogenous. Assume that in the second period the government decides to subsidize labor income at rate $\tau = 100\%$, that is for every NOK you earn the government gives you one NOK in addition, so that the after-tax wage for the consumer in the second period is doubled. How does labor supply and consumption in both periods change? How do the wages paid by the firm in the two periods change? Explain the intuition of your findings. (hint: the answer does not involve any additional math.)