UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4310 – Macroeconomic Theory**

Date of exam: Monday, November 30, 2015 **Grades are given: December 21, 2015**

Time for exam: 2.30 p.m. – 5.30 p.m.

The problem set covers 12 pages (incl. cover sheet)

Resources allowed:

- No resources allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences)

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Final Exam
ECON 4310, Fall 2015

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	**Max**
Exercise A		**60**
Exercise B		**60**
Exercise C		**60**
Σ		**180**

Grade: __________

Exercise A:
Short Questions (60 Points)

Answer each of the following short questions on a seperate answer sheet by stating as a first answer True/False and then give a short but instructive explanation. You can write, calculate, or draw to explain your answer. You only get points if you have stated the correct True/False *and* provided a correct explanation to the question. We will not assign negative points for incorrect answers.

Exercise A.1: (10 Points) The Intertemporal Elasticity of Substitution

Consider the optimal intertemporal consumption choice of a household in discrete and infinite time. The representative household has preferences $U = \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta \in (0,1)$ is the discount factor and the momentary utility function is

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}, \quad \theta > 1.$$

The optimal behavior is characterized by the following consumption Euler equation

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1}{1+r_{t+1}}.$$

The Intertemporal Elasticity of Substitution (IES)

$$\text{EIS} \equiv \frac{\partial \log(c_{t+1}/c_t)}{\partial \log(1+r_{t+1})},$$

does *not* depend on the discount factor β for this momentary utility function. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.2: (10 Points) A Solow Model without productivity growth

Consider the following version of the Solow model, with an alternative production function specified in (2), a growing population, L_t, and a constant saving rate, s, as described by the following equations

$$K_{t+1} = sY_t + (1-\delta)K_t \tag{1}$$

$$Y_t = \left[K_t^\rho + L_t^\rho\right]^{1/\rho}, \quad \rho > 0, \tag{2}$$

$$L_{t+1} = (1+n)L_t,\ L_0 > 0, A = 1,$$

where $0 < \delta < 1$ is the depreciation rate of physical capital, and $n + \delta > s$. (Hint: you will have to repeatedly use the fact that

$$x_t^{-1}\left[K_t^\rho + L_t^\rho\right]^{1/\rho} = \left[K_t^\rho x_t^{-\rho} + L_t^\rho x_t^{-\rho}\right]^{1/\rho},$$

in your explanation.)

The steady-state level of the capital stock per capita, $k_t \equiv K_t/L_t$, is given by

$$k^\star = \left[\left(\frac{n+\delta}{s}\right)^\rho - 1\right]^{-1/\rho}.$$

True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.3: (10 Points) War spending in the Ramsey model

Consider the dynamic equilibrium equations of the Ramsey model

$$\frac{c_{t+1}}{c_t} = \left[\beta(1+\alpha k_{t+1}^{\alpha-1}-\delta)\right]^{1/\theta}, \quad c_t \equiv C_t/(A_t L_t),$$
$$k_{t+1} - k_t = k_t^{\alpha} - \delta k_t - c_t - \tau, \quad k_t \equiv K_t/(A_t L_t),$$

where K_t is the aggregate capital stock, $\alpha k_{t+1}^{\alpha-1} - \delta$ the interest rate, A_t is the state of technology, L_t the growing size of the population, C_t aggregate consumption, $\alpha \in (0,1)$ the capital income share in the economy, $\delta \in (0,1)$ the depreciation rate of physical capital, $\beta \in (0,1)$ is the subjective discount factor, and $1/\theta$ the intertemporal elasticity of substitution. Finally, τ is the lump-sum tax which finances the constant government expenditure according to a balanced budget,

$$\tau = G.$$

Suppose that the economy is in the steady-state. Consider now a temporary and unexpected increase of the lump-sum tax, τ, to finance an unexpected war.

As a consequence, consumption per efficiency unit, c_t, will jump upwards on impact and then fall in the transition back to the steady-state. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.4: (10 Points) Income, substitution and wealth effect

Consider a representative consumer who lives for only two periods denoted by $t = 1, 2$. The consumer is born in period 1 without any financial assets and leaves no bequests or debt at the end of period 2. The consumer's labor income is w_t in each period and her preferences over consumption can be represented by the utility function

$$U(c_1, c_2) = u(c_1) + \beta u(c_2), \quad 0 < \beta < 1, \tag{3}$$

where the momentary utility function is given by

$$u(c) = \begin{cases} \frac{c^{1-\theta}-1}{1-\theta}, & \theta \neq 1. \\ \log(c), & \theta = 1, \end{cases}$$

with $\theta > 0$. The optimal consumption in period 1 is given by

$$c_1 = \frac{1}{1 + \beta^{1/\theta}(1+r)^{1/\theta - 1}} \left(w_1 + \frac{w_2}{1+r} \right).$$

Now, consider the effect of an increase in the gross real interest rate $1 + r$ on the optimal first period consumption. The overall effect of this increase is always strictly positive if the labor income in the second period is zero, $w_2 = 0$. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.5: (10 Points) Optimal policy, Laffer curve

Suppose the aggregate labor supply, $h(\tau)$, of an economy as a function of the labor income tax rate, τ, is given by

$$h(\tau) = [(1-\tau)w]^{\varphi}\,.$$

where φ is the Frisch elasticity of labor supply. Suppose that the government must raise a level of tax revenue to finance an expenditure $g^{\star}$, where

$$0 < g^{\star} < \bar{\tau} w h(\bar{\tau}),$$

and $\bar{\tau}$ is the tax rate associated with the top of the Laffer curve.

The optimal labor income tax rate to finance $g^{\star}$ is only one, and it is $\tau = \bar{\tau}$.
True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.6: (10 Points) Optimal policy: fully-funded pension system

Consider the simple two period life-cycle overlapping generations model. The representative agent lives for two period $t = 1, 2$, and has a labor income $w_1 > 0$ in the first period, no labor income in the second period, $w_2 = 0$, and can save across periods. The agent's preferences can be represented by the utility function

$$U(c_1, c_2) = \log(c_1) + \beta \log(c_2), \quad 0 < \beta < 1,$$

and she is subject to the lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = w_1,$$

where $r > 0$ is a given exogenous interest rate. The government considers the introduction of a compulsory fully-funded pension system, such that the agent is obliged to contribute a lump-sum $\tau > 0$ in the first period, and receives a pension

$$P = (1 + \tilde{r})\tau,$$

in the second period. Moreover, suppose the government has better investment opportunities than the agent such that $\tilde{r} > r$.

The introduction of the pension scheme will never affect the optimal consumption choice of the agent. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise B:
Long Question (60 Points)

A comparison of the Solow model and the Ramsey growth model

Consider a closed economy without exogenous technology and population growth, where firms produce a generic good Y_t with the production function

$$Y_t = F(K_t, L) = K_t^{\alpha} L^{1-\alpha}, \quad 0 < \alpha < 1,$$

where K_t is aggregate capital and L is the number of workers in the economy. The law of motion for aggregate capital is given by

$$K_{t+1} = (1-\delta) K_t + I_t, \quad K_0 > 0, \tag{4}$$

where I_t denotes aggregate investment, and $0 < \delta < 1$ the depreciation rate. For simplicity, let aggregate labor supply (population) be equal to one, $L = 1$, such that consumption per worker, c_t, is the same as aggregate consumption, $C_t = c_t = c_t L$.

Consider now two different models. The Solow model where agents have a constant savings rate, s, such that

$$I_t = sY_t.$$

And the Ramsey growth model where household utility is maximized

$$\sum_{t=0}^{\infty} \beta^t u(C_t), \tag{5}$$

such that aggregate investment (savings) is endogenous

$$I_t = Y_t - C_t.$$

In both models markets are competitive, thus input factors K_t and L are paid their marginal products.

(a) (10 Points) Compute the interest rate in this Solow model's stable steady state. (hint: compute the stable aggregate steady-state capital stock first.)

(b) (10 Points) In the Ramsey growth model households maximize lifetime utility in Equation (5) with respect to C_t and K_{t+1} and subject to the law of motion of capital stated in Equation in (4). Taking into account the functional form of output, Y_t, and investment, I_t, and that $L = 1$, write up the Lagrangian of this maximization problem and derive the following optimality conditions of the Ramsey growth model

$$\frac{u'(C_t)}{u'(C_{t+1})} = \beta \left[1 + \alpha (K_{t+1})^{\alpha-1} - \delta\right]$$
$$K_{t+1} - K_t = K_t^{\alpha} - \delta K_t - C_t.$$

The first optimality condition is the model's Euler equation and the second the resource constraint.

(c) (5 Points) Compute the interest rate in the Ramsey model in a steady-state. (hint: you can solve this part even if you were not able to solve part (b).)

(d) (10 Points) Does the interest rate in the Solow model depend on the saving rate s in the stable steady-state? Is the intereste rate increasing or decreasing in s or independent from s. Why? (hint: you can try to answer this question even though you were not able solve previous parts.)

(e) (10 Points) Does the interest rate in the Ramsey model depend on the discount factor β in a steady-state? Is the interest rate increasing or decreasing in β or independent from β. Why? (hint: you can try to answer this question even though you were not able solve previous parts.)

(f) (5 Points) Compute the saving rate $\bar{s}$ which gives the same steady-state capital stock in the Solow model as in the Ramsey growth model. Is this saving rate, $\bar{s}$, increasing or decreasing in the discount factor β?

(g) (10 Points) Compute the the saving rate $\hat{s}$ which gives the same interest rate in the Solow model as in the Ramsey growth model. Is this saving rate increasing or decreasing in the depreciation rate δ?

Exercise C:
Long Question (60 Points)

Consumption response to income shocks

Consider a household decision problem under uncertainty, when preferences are linear-quadratic:

$$u(c_t) = c_t - \frac{b}{2}c_t^2, \quad b \geq 0.$$

The household lives from period 0 to period $T < +\infty$ and discounts the future at rate $\beta = 1$. Assume $1 + r = 1/\beta = 1$, that is $r = 0$. The lifetime budget constraint of the household viewed from period 0 reads

$$(1+r)\,a_0 + \sum_{t=0}^{T} \frac{I_t}{(1+r)^t} = \sum_{t=0}^{T} \frac{c_t}{(1+r)^t}, \tag{6}$$

and viewed from period 1

$$(1+r)\,a_1 + \sum_{t=1}^{T} \frac{I_t}{(1+r)^t} = \sum_{t=1}^{T} \frac{c_t}{(1+r)^t}, \tag{7}$$

where I_t is income in period t which is uncertain in any previous period $s \leq t$ and only learned in period t. Expected income (or, consumption) in period t viewed from period $s \leq t$ equals $E_s[I_t]$ (or, $E_s[c_t]$), and the Euler equation is given by

$$u'(c_t) = \beta(1+r)E_t\left[u'(c_{t+1})\right], \quad \forall t \leq T-1. \tag{8}$$

(a) (10 Points) Under the given assumptions, (i) show that the Euler equation can be written as

$$c_t = E_t\left[c_{t+1}\right], \quad \forall t \leq T-1, \tag{9}$$

and, (ii) give an interpretation (just one sentence!) of Equation (9).

(b) (10 Points) Note that Equation (9) together with the law of iterated expectations implies that

$$E_s\left[c_t\right] = c_s, \quad \forall s \leq t.$$

Now, assume that the household expects to not pay any taxes such that income I_t is equal to wage income w_t in any period t. Take expectations on the lifetime budget constraints and show that consumption in period 0 and 1 are given by

$$c_0 = (T+1)^{-1}\left(a_0 + \sum_{t=0}^{T} E_0\left[I_t\right]\right), \quad E_0[I_t] = E_0[w_t],$$

and

$$c_1 = T^{-1}\left(a_1 + \sum_{t=1}^{T} E_1\left[I_t\right]\right), \qquad E_1[I_t] = E_1[w_t],$$

respectively.

(c) (10 Points) Now assume that the household knows (already in period 0) that from period 1 to period T the government levies taxes g, so that income is $I_t = w_t - g$ from period 1 onwards. Derive again consumption in period 0 and 1.

(d) (10 Points) Assume now that in period 0 the household expects to never pay taxes. In period 1 the household is surprised since the government levies now taxes g from period 1 to period T, so that income $I_t = w_t - g$ from period 1 to T. Derive consumption in period 0 and 1.

(e) (10 Points) Assume now that in period 0 the household expects to never pay taxes. In period 1 the household is surprised since the government levies now taxes g but only in period 1. From period 2 to period T no taxes are levied.

(f) (10 Points) Explain your results and the differences in consumption for the different scenarios in parts (c), (d), and (e), relative to part (b).