UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam: **ECON4310 – Macroeconomic Theory**

Date of exam: Wednesday, January 11, 2017

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 13 pages (incl. cover sheet)

Resources allowed:

- No written or printed resources – or calculator - is allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences)

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Final Exam II
ECON 4310, Fall 2016

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	**Max**
Exercise A		**60**
Exercise B		**60**
Exercise C		**60**
Σ		**180**

Grade: __________

Exercise A:
Short Questions (60 Points)

Answer each of the following short questions on a seperate answer sheet by stating as a first answer True/False and then give a short but instructive explanation. You can write, calculate, or draw to explain your answer. You only get points if you have stated the correct True/False *and* provided a correct explanation to the question. We will not assign negative points for incorrect answers.

Exercise A.1: (10 Points) Value of firm ownership in a static competitive equilibrium

Consider a static economy with many households. Households consume and supply labor to the market inelastically. Assume each household supplies 1 unit of labor. Households do not have initial wealth.

Each household owns a firm which produces the same output good y, with following production technology

$$y = An,$$

where A is technology, n is labor hired from the competitive labor market. Assume the technology is the same for all households.

The household's utility function is $\log(c)$. In our static economy, the household will consume everything he/she has, including any labor income and any business income.

Households then have different consumption levels.

True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.2: (10 Points) Solow model with worker heterogeneity

Consider the following version of the Solow model with two types of workers, type 1 with productivity A_1 and type 2 with productivity A_2. The number of workers are given by L_1 and L_2, respectively. There are no population growth or technology growth. The production function is specified in (2), and the economy is described by the following equations

$$K_{t+1} = sY_t + (1-\delta)K_t \tag{1}$$

$$Y_t = \left[K_t^{\rho} + (A_1 L_1)^{\rho} + (A_2 L_2)^{\rho}\right]^{1/\rho}, \quad \rho > 0, \tag{2}$$

where $0 < \delta < 1$ is the depreciation rate of physical capital, and the saving rate s is constant. A_1, A_2 and L_1, L_2 are also constant. Denote the share of type 1 workers as $\theta \equiv \frac{L_1}{L} = \frac{L_1}{L_1+L_2}$.

The steady-state level of the capital stock per capita, $k_t \equiv K_t/L$, is described by

$$\delta k^{\star} = s\left[(k^{\star})^{\rho} + (A_1(1-\theta))^{\rho} + (A_2\theta)^{\rho}\right]^{1/\rho}.$$

True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.3: (10 Points) Time series of consumption growth and interest rate

Consider the optimal intertemporal consumption choice of a household in discrete and infinite time. The representative household has preferences $U = \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta \in (0, 1)$ is the discount factor and the momentary utility function is

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}, \quad \theta > 1.$$

Suppose that the time series for r_{t+1} vary and are exogenously given.

The optimal behavior for the household is characterized by the following consumption Euler equation

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1}{1 + r_{t+1}}.$$

If you run an Ordinary Least Square regression of the time series for consumption growth, $\log(c_{t+1}/c_t)$, on the time series of interest rate, $\log(1 + r_{t+1})$, the slope coefficient would be θ.

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.4: (10 Points) Capital accumulation equation in the Solow model

Consider the Solow model with exogenous technological progress, A_t, a fixed saving rate, s, and a constant labor force, L, described by the following equations:

$$
\begin{aligned}
K_{t+1} - K_t &= sY_t - \delta K_t \\
Y_t &= K_t^{\alpha}(A_t L)^{1-\alpha}, \quad 0 < \alpha < 1, \\
A_{t+1} &= (1+g)A_t, \quad A_0 > 0,
\end{aligned}
$$

where $0 \leq \delta \leq 1$ is the depreciation rate of physical capital.
The capital accumulation equation in terms of efficiency units is described by the following equation:

$$k_{t+1} = \frac{1}{1+g}(sk_t^{\alpha} + (1-\delta)k_t)$$

True or false?

Your Answer:

True: ☐ False: ☐

Don’t forget the explanation!

Exercise A.5: (10 Points) Optimal policy, Laffer curve

Suppose the aggregate labor supply, $h(\tau)$, of an economy as a function of the labor income tax rate, τ, is given by

$$h(\tau) = [(1-\tau)w]^{1/3}.$$

The top of the Laffer curve is given by $\bar{\tau} = 1/3$. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.6: (10 Points) A peculiar Fiscal Policy

Consider a small open economy populated by non-overlapping generations living one period and by an infinitely lived government, endowed with asset $a_0 = A > 0$ at time $t = 0$. Each generation is subject to the private budget constraint:

$$c_t = w_t + T_t$$

and the government is subject to the period-by-period gov. budget constraint:

$$a_{t+1} = (1+r)a_t - T_t$$

where c_t is private consumption, w_t is exogenous private income, a_t is government's net saving, T_t is a public transfer from the government to the generation t, and r is the constant interest rate.
Wage grows at rate g such that $w_{t+1} = (1+g)w_t$.
The government follows the following fiscal rule:

$$\frac{a_t}{w_t} = \frac{A}{w_0} \quad \forall t.$$

If $r > g$ then transfers T_t are negative $\forall t$. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise B:
Long Question (60 Points)

A real business cycle model

Consider a representative household of a closed economy. The household has a planning horizon of two periods and is endowed with the following preferences over consumption, c,

$$U = u(c_1) + \beta \mathrm{E} u(c_2(s_2)),$$

with the following marginal utility

$$u'(c) = c^{-1}, \text{ (log-utility)}.$$

The variable s_2 denotes the state of the economy in the second period which follows the stochastic process

$$s_2 = \begin{cases} s_G, \text{ with prob. } p \\ s_B, \text{ with prob. } 1-p, \end{cases}$$

and the household conditions the consumption, $c_2(s_2)$, in the second period on the state, s_2. Assume the household's labor supply is exogenous and always equal to 1.

Labor market assumptions:

Assume that in each period and in each state of the economy, s_t, there is a linear (in labor n_t) production technology of the form

$$y_t(s_t) = A_t(s_t) n_t(s_t),$$

and the labor market is assumed to be competitive. Assume the labor productivity in the first period is given by $A_1 = A$, and the labor productivity is higher in the good state of the second period,

$$A_2(s_G) = A + A(1-p)\sigma > A_2(s_B) = A - Ap\sigma, \quad \sigma > 0, A > 0, 0 < p < 1,$$

than in the bad state of the second period. Notice that the expected productivity in the second period is the same as the productivity in the first period, $E[A_2(s_2)] = A_1 = A$. The wages are denoted as w_1, $w_2(s_G)$, and $w_2(s_B)$.

Asset market assumptions:

Assume the household does have access to a risk-free asset, a_2, and the associated interest rate is denoted as r_2.

(a) (5 Points) Find the equilibrium wages, w_1, $w_2(s_G)$, and $w_2(s_B)$.

(b) (5 Points) Write down the state-by-state budget constraints for the household.

(c) (10 Points) Let $(\lambda_1, \lambda_2(s_G), \lambda_2(s_B))$ denote the Lagrange multipliers of the state-by-state budget constraints. State the representative agent's Lagrangian. (Note that the expected utility for the second period is the summation of utility across good and bad states, weighted by probability, i.e., $\mathrm{E}u(c_2(s_2)) = pu(c_2(s_G)) + (1 - p)u(c_2(s_B))$.)

(d) (10 Points) Derive the optimality conditions with respect to consumption, $(c_1, c_2(s_G), c_2(s_B))$ and savings, a_2 by using multipliers.

(e) (10 Points) Derive the stochastic consumption Euler equation (only involves with $c_1, c_2(s_2), \beta$ and r_2 and No multipliers).

(f) (10 Points) Show that $u'(w_1) \leq \mathrm{E}[u'(w(s_2))]$.

(Hint: use the Jensen's inequality. The Jensen's inequality says that, if we have a random variable x_2 with its mean Ex_2 at x_M, then for any convex function $f(.)$, we should have the inequality $f(x_M) \leq Ef(x_2)$.)

(g) (10 Points) Now assume the interest rate is exogenously given by $r_2 = \frac{1}{\beta} - 1$. What would happen to the household's optimal choice of a_2, i.e., $a_2 < 0$, or $a_2 = 0$, or $a_2 > 0$?

Is the expected consumption in the second period higher or lower than that in the first period, i.e., $Ec_2(s_2) < c_1$, or $Ec_2(s_2) = c_1$, or $Ec_2(s_2) > c_1$? Why?

Exercise C:
Long Question (60 Points)

A comparison of the Ramsey model and the OLG model

Consider a closed economy without exogenous technology and population growth, where firms produce a generic good Y_t with the production function

$$Y_t = F(K_t, L) = K_t^{\alpha} L^{1-\alpha}, \quad 0 < \alpha < 1,$$

where K_t is aggregate capital and L is the number of workers in the economy. Capital depreciates at the rate $0 < \delta < 1$. For simplicity, let aggregate labor supply (population) be equal to one, $L = 1$, such that per capita variables, $x_t = X_t/L$, are the same as aggregate variables, $X_t = x_t = X_t L$.

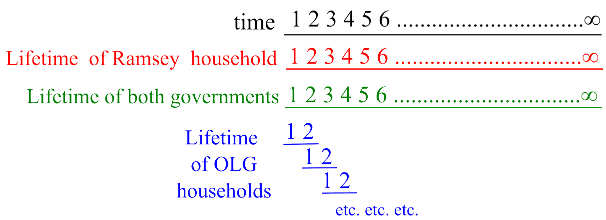
Consider now two different models. The Ramsey model where households are infinitely lived; and the OLG model where households live two periods, and where in each period two different generations are alive (young and old).
In both models agents chose consumption and asset holdings in order to maximize discounted lifetime utility:

$$\text{Ramsey:} \quad \sum_{t=1}^{T} \beta^t u(c_t)\,; \quad \text{OLG:} \quad u(c_t^y) + \beta u(c_{t+1}^o); \quad u(c) = \frac{c^{1-\theta} - 1}{1-\theta}, \quad \theta > 0$$

Each period t, the household is endowed with: the wage rate w_t (the labor supply is exogenously set to 1 unit) net of lump-sum tax τ_t, and the saving from the previous period a_t, plus the interest on it $r_t a_t$. Each period t, the household must decide how much to consume c_t, and how much to save a_{t+1}.
At birth each household is endowed with zero assets (in Ramsey $a_1 = 0$; in OLG $a_t = 0$ for an household born at t).
In both models markets are competitive, thus the prices of input factors are equal to their marginal products.
Both models are also populated by an infinitely-lived government who uses taxes τ_t and public debt D_t to finance government expenditure G_t. The government is subject to the period-by-period government budget constraint:

$$G_t = \tau_t + D_{t+1} - (1 + r_t) D_t$$

that holds $\forall t$ and the initial public debt is zero, i.e. $D_1 = 0$.
The timing is summarized in the following timeline:



(a) (5 Points) Write the period-by-period private budget constraint in the Ramsey model. (Hint: you need to write only one equation that holds $\forall t$)

(b) (5 points) Focus on the OLG model. For the young generation at time t, denote the savings as a^y_{t+1}, wage as w^y_t and consumption as c^y_t; for the old at time $t+1$, denote the wage as w^o_{t+1}, consumption as c^o_{t+1} and possible savings as a^o_{t+2}.
Write the period-by-period private budget constraints of the household born in period t. (Hint: you need two constraints, one of the household when young, and one of the same household when old.)

(c) (10 Points) In the Ramsey growth model households maximize lifetime utility with respect to c_t and a_{t+1} and subject to the period-by-period budget constraint derived at point (a). Write up the Lagrangian of this maximization problem and derive the following consumption Euler Equation of the Ramsey model:

$$\frac{c_{t+1}}{c_t} = [\beta(1 + r_{t+1})]^{1/\theta}$$

(d) (10 Points) In the OLG model, households born in t maximize lifetime utility with respect to c^y_t, a^y_{t+1}, c^o_{t+1}, and a^o_{t+2}, and subject to the period-by-period budget constraints derived at point (b), and the constraint $a^o_{t+2} \geq 0$. Without any calculation, find the optimal a^o_{t+2}. Then, write up the Lagrangian of this maximization problem and derive the following consumption Euler Equation of the OLG model:

$$c^o_{t+1}/c^y_t = [\beta(1 + r_{t+1})]^{1/\theta}.$$

(e) (15 Points) Suppose that the sequence $\{\tau_t, G_t\}_{t=0}^{\infty}$ satisfies all the constraints and conditions imposed so far.
At t_1 the government decides a one-time unexpected increase in government expenditures ΔG_{t_1}. This increase can be financed with higher taxes $\Delta\tau_{t_1}$ OR higher debt ΔD_{t_1+1}.
Assume the households have perfect foresight, and anticipate the government budget constraint.
Is the household's optimal consumption decision different depending on whether ΔG_t is fully financed via the tax increase, or fully financed via a debt increase above?

- Answer the question above for the Ramsey model.
- Answer the question above for the OLG model.
- Motivate your answers.

(f) (5 Points) Derive the equilibrium prices of labor (w_t) and the equilibrium price of capital (R_t) in both models.
(hint: in equilibrium aggregate demand and supply of labor are equal.)

(g) (10 Points) Focus now on the Ramsey model. Combine the period-by-period private budget constraint, and the period-by-period government budget constraint together with the market clearing in the capital market, $a_t = k_t + D_t \quad \forall t$, to show that the equation describing capital accumulation in the Ramsey model is:

$$k_{t+1} - k_t = k_t^{\alpha} - \delta k_t - c_t - G_t$$

(Hint: do it step by step: (1) start with the capital market clearing condition; (2) use appropriately the period-by-period private budget constraint derived in (a); (3) use the period-by-period government budget constraint; (4) use the results in (f) and $R_t = r_t + \delta$.)