i Instructions

ECON4310 – Macroeconomic Theory - postponed

This is some important information about the postponed exam in ECON4310. Please read this carefully before you start answering the exam.

Date of exam: Monday, January 7, 2019

Time for exam: 09.00 a.m. – 12.00 noon

The problem set: The problem set consists of 3 exercises, with several subexercises. They count as indicated.

Sketches: You may use sketches on all questions. You are to use the sketching sheets handed to you. You can use more than one sketching sheet per question. See instructions for filling out sketching sheets on your desk. It is very important that you make sure to allocate time to fill in the headings (the code for each problem, candidate number, course code, date etc.) on the sheets that you will use to add to your answer. You will find the code for each problem under the problem text. You will NOT be given extra time to fill in the headings on the sketching sheets.

Access: You will not have access to your exam right after submission. The reason is that the sketches with equations and graphs must be scanned in to your exam. You will get access to your exam within 2-3 days.

Resources allowed: No written or printed resources - or calculator - is allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences).

Grading: The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

1.1 Exercise 1.1: 10 Points

Static Competitive Equilibrium

Consider a static economy with a representative consumer that has the following preferences over consumption, *c*, and labor supply, *h*,

$$u(c,h) = log(c) + log(1-h),$$

and is subject to the budget constraint

$$c = wh,$$

where *w* is the wage rate per unit of labor supplied. The optimal labor supply is then independent of the wage rate,

$$h = 1/3.$$

True or false?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
Use the following code:

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1.2 Exercise 1.2: 10 Points

Fiscal Policy

Consider a small open economy populated by non-overlapping generations living one period and by an infinitely lived government, endowed with asset $a_0 = A > 0$ at time $t = 0$. Each generation is subject to the private budget constraint:

$$c_t = w_t + T_t$$

and the government is subject to the period-by-period gov. budget constraint:

$$a_{t+1} = (1+r)a_t - T_t$$

where c_t is private consumption, w_t is exogenous private income, a_t is government's net saving, T_t is a public transfer from the government to the generation t, and r is the constant interest rate.

Wage grows at rate g such that $w_{t+1} = (1+g)w_t$.

The government follows the following fiscal rule:

$$a_t = (1+x)^t A \quad \forall t$$

If $r < g$ then transfers T_t are negative $\forall t$.

True or false?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?

Use the following code:

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1.3 Exercise 1.3: 10 Points

Permanent Technology shocks in Real business cycle model and Consumption Response

Consider a simple two-period model of labor supply, as we saw in lectures, where we assume that utility is separable in consumption and labor supply:

$$\max_{\{c_0,c_1,h_0,h_1,a_1\}} \log c_0 - \phi\frac{h_0^{1+\theta}}{1+\theta} + \beta[\log c_1 - \phi\frac{h_1^{1+\theta}}{1+\theta}]$$
$$s.t.$$
$$c_0 + a_1 = w_0 h_0 + (1+r_0)a_0$$
$$c_1 = w_1 h_1 + (1+r_1)a_1$$

for given $a_0 = 0$. Assume r_0, r_1 are exogenously given. We know the household has the following intertemporal labor supply considion:

$$\beta\frac{\phi h_1^\theta}{\phi h_0^\theta} = \frac{w_1}{(1+r_1)w_0},$$

and the solution for h_0 is given by

$$\phi h_0^{1+\theta}\left[1 + \left(\frac{w_1}{(1+r_1)w_0}\right)^{1+\frac{1}{\theta}}\beta^{-\frac{1}{\theta}}\right] = (1+\beta).$$

Suppose there is a permanent change to wages at the beginning of time 0: both wages in the first and second period increase by 10%. Then this household will take advantage of this opportunity and increase consumption in period 0, c_0, by 10%.

True or false?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?

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1.4 Exercise 1.4: 10 Points

Optimal policy, Laffer curve
Suppose the aggregate labor supply, $h(\tau)$, of an economy as a function of the labor income tax rate, τ, is given by

$$h(\tau) = [(1-\tau)w]^{1/2}.$$

The top of the Laffer curve is given by $\bar{\tau} = 1/2.$

True or false?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
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2 Consider a representative consumer who lives for only two periods denoted by t = 1, 2. The consumer is born in period 1 without any financial assets and leaves no bequests or debt at the end of period 2, such that she is subject to the period-by-period budget constraints

$$c_1 + s = w_1$$
$$c_2 = w_2 + (1+r)s,$$

where s denotes the amount of savings. The consumer's labor income is w_t in each period and her prefeences over consumtion can be represented by the utility function

$$U(c_1, c_2) = log(c_1) + \beta\, log(c_2), \quad 0 < \beta < 1. \qquad (1)$$

For the moment we abstract from the production side of the economy and simply assume that the consumer can borrow and lend consumption across periods at the given real interest rate, $r > 0$. We assume implicitly that the depreciation rate of capital is zero, $\delta = 0.$

1 Exercise 2.1: 20 Points

Write down the consumer's net present value budget constraint, and show that the optimal consumption in period 1 is given by

$$c_1 = \tfrac{1}{1+\beta}\left(w_1 + \tfrac{w_2}{1+r}\right).$$

State also the optimal savings.

Fill in your answer here and/or on sketching paper

Maximum marks: 20

Attaching sketches to this question?
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2 Exercise 2.2: 10 Points

In the above analysis you will have found that the optimal consumption growth over the life-cycle satisfies the Euler equation

$$\tfrac{c_2}{c_1} = \beta(1+r).$$

What is the elasticity of interremporal substitution (EIS)

$$\text{EIS} = \frac{\partial \log(c_2/c_1)}{\partial \log(1+r)},$$

of this model specification then?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
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Exercise 2.3: 10 Points

We now turn from the representative consumer behavior to the economy as a whole. Suppose that this economy is populated by an infinite sequence of overlapping generations that live for two periods. Each generation is of size, Lt, where

$$L_{t+1} = (1+n)L_t, n > 0, L_0 > 0,$$

and an individual's old-age income is assumed to be zero,

$$w_2 = 0.$$

There is a production sector that combines aggregate physical capital, Kt, and labor, Lt, according to the technology

$$Y_t = F(K_t, Y_t L_t) = K_t^\alpha (A_t L_t)^{1-\alpha}, \quad 0 < \alpha < 1,$$

to produce output Yt. Markets are competitive such that wage rate and the rental rate of capital are given by their marginal product

$$w_t = (1-\alpha) A_t k_t^\alpha, \quad k_t \equiv K_t/(A_t L_t),$$
$$r_t = \alpha k_t^{\alpha-1},$$

and $A_{t+1} = (1+g)A_t, g > 0, A_0 > 0.$ Young agents save by buying unit claims to next period's capital stock, such that capital market clearing requires that the aggregate savings of the young, St, corresponds to the next peirod physical capital stock

$$S_t \equiv s_t L_t = K_{t+1}, \qquad (2)$$

where st denotes the savings per capita of the current young.

Compute the aggregate savings, S_t, in this economy and use the capital market clearing condition in Equation (2) to characterize the future capital stock $K_{t=1}$ as a function of the current A_t, k_t and L_t. (Hint: if you were not able to solve for individual consumption and savings in Exercise 2.1, you can assume that a constant fraction of the wage income is saved by each household,

$$s_t = \gamma w_t, \quad 0 < \gamma < 1,$$

to make further progress.)

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
Use the following code:

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Exercise 2.4: 10 Points

Derive the law of motion for the capital stock per efficiency unit, k_{t+1} as a function of k_t, sketch it in a diagram with k_{t+1} on the vertical and k_t on the horizontal axis, and mark the stable steady state in the diagram (you do not have to compute the steady state).

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
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Exercise 2.5: 10 Points

Suppose the economy is in the stable steady state. Suddenly, in period t_0, due to a natural desaster half of the aggregate capital stock is destroyed. Sketch the dynamics of the capital stock per efficiency unit caused in response to this unexpected shock. Also, in a separate time diagram, sketch the dynamics of the logarithm of the wage rate over time. Be explicit in the diagrams whether a variable falls/increases by more or less than half on impact.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
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Consider a representative household of a closed economy. The household has a planning horizon of two periods and is endowed with the following preferences over consumption, *c*,

$$U = u(c_1) + \beta E u(c_2(s_2)),$$

with the following marginal utility

$$u'(c) = c^{-\gamma}, \gamma \geq 1.$$

The variable s_2 denotes the state of the economy in the second period which follows the stochastic process

$$s_2 = \begin{cases} s_G, \text{ with prob. } p \\ s_B, \text{ with prob. } 1-p, \end{cases}$$

and the household conditions the consumption, $c_2(s_2)$, in the second period on the state, s_2. Assume the household's labor supply is exogenous and always equal to 1.

Labor market assumptions:

Assume that in each period and in each state of the economy, s_t there is a linear (in labor n_t) production technology of the form

$$y_t(s_t) = A_t(s_t) n_t(s_t),$$

and thelabor market is assumed to be competitive. Assume the labor productivity in the first period is given by A_1 = *A*, and the labor productivity is higher in the good state of the second period,

$$A_2(s_G) = A + A(1-p)\epsilon > A_2(s_B) = A - Ap\epsilon, \quad \epsilon > 0, A > 0, 0 < p < 1,$$

than in the bad state of the second period. The wages are denoted as w_1, $w_2(s_G)$, and $w_2(s_B)$.

Asset market assumptions:

Assume the household does have access to a risk-free asset, a_2, and the associated interest rate is denoted as r_2.

1

Exercise 3.1: 5 Points

Find the equilibrium wages, $w_1, w_2(s_G)$, and $w_2(s_B)$, an show that the expected wages in the second period is the same as wage in the first period.

Fill in your answer here and/or on sketching paper

Maximum marks: 5

Attaching sketches to this question?
Use the following code:

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Exercise 3.2: 5 Points

Write down the state-by-state budget constraints for the household.

Fill in your answer here and/or on sketching paper

Maximum marks: 5

Attaching sketches to this question?
Use the following code:

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Exercise 3.3: 10 Points

Let $(\lambda_1, \lambda_2(s_G), \lambda_2(s_B))$ denote the Lagrange multipliers of the state-by-state budget constraints. State the representative agent's Lagrangian. (Note that the expected utility for the second period is the summation of utility across good and bad states, weighted by probability, i.e., $Eu(c_2(s_2)) = pu(c_2(s_G)) + (1-p)u(c_2(s_B)).$)

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
Use the following code:

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Exercise 3.4: 10 Points

Derive the stochastic consumption Euler equation (only involves with $c_1, c_2(s_2), \beta$and r_2 and No multipliers).

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
Use the following code:

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Exercise 3.5: 10 Points

For Exercise 3.5, assume that the asset a_2 is available in zero supply.

What is the household's optimal choice of a_2 in the equilibrium? What are the household's optimal choices of consumption? Can the household fully smooth consumption? i.e., are $c_1, c_2(s_G)$ and $c_2(s_B)$ equal?
Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
Use the following code:

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Exercise 3.6: 10 Points

For Exercise 3.6, assume that the asset a_2 is available in zero supply.

Assume now for this part that $A_1 = 2$ in period 1, $\epsilon = 1$, and $\beta = 1/2, \gamma = 1$ and $p = 1/2$. What is the gross interest rate 1 + *r* in equilibrium?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
Use the following code:

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Exercise 3.7: 10 Points

For Exercise 3.7, assume that the asset a_2 is available in zero supply.

Is the equilibrium interest rate r_2 higher or lower than $r_{RN} \equiv \frac{1}{\beta} - 1$? Why?

(Hint: do it step by step: (1) use the budget constraint to link consumption and wages; (2) use the Euler equation and the result, $u'(w_1) \leq E[u'(w(s_2))]$, which comes from the Jensen's inequality.)

Fill in your answer here and/or on sketching paper

Maximum marks: 10

Attaching sketches to this question?
Use the following code:

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