

## **i** Candidate instructions

### **ECON4310**

This is some important information about the postponed exam in ECON4310. Please read this carefully before you start answering the exam.

**Date of exam:** Tuesday, January 14, 2020

**Time for exam:** 09.00 a.m. - 12.00 noon (3 hours)

**The problem set:** The problem set consists of 4 questions with several sub-questions. They count as indicated.

**Sketches:** You may use sketches on all questions. You are to use the sketching papers handed to you. You can use more than one sketching sheet per question. See instructions for filling out sketching papers on "Scantron information" below. It is very important that you make sure to allocate time to fill in the headings (the code for each problem, candidate number, course code, date etc.) on the sheets that you will use to add to your answer. You will find the code for each question under the question text. You will NOT be given extra time to fill out the "general information" on the sketching.

**Access:** You will not have access to your exam right after submission. The reason is that the sketches with equations and graphs must be scanned in to your exam. You will get access to your exam within 2-3 days.

**Resources allowed:** No written or printed resources - or calculator - is allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences).

**Grading:** The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

1      **The Solow Model**

Consider an economy on its balanced growth path (in steady state) that is characterized by the Solow growth model and by the fact that there is technological progress and no population growth. Now suppose there is a one-time jump in the number of workers.

(a)    **(a) - 10 points**

At the time of the jump, does output per unit of effective labor rise, fall, or stay the same? Why? Explain your answer.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(b)    **(b) - 10 points**

After the initial change (if any) in output per unit of effective labor when the new workers appear, is there any further change in output per unit of effective labor? If so, does it rise or fall? Why? Explain your answer.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(c)    **(c) - 10 points**

Once the economy has again reached a balanced growth path, is output per unit of effective labour higher, lower, or the same as it was before the new workers appeared? Why? Explain your answer.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

## 2 Savings and Uncertainty

Consider a model in which there are two periods ( $t = 1, 2$ ) and a unit mass of identical agents. In period 2 there are two states, denoted by  $s_G$  and  $s_B$ . The state turns out to be  $s_G$  with probability  $p \in (0, 1)$  and thus state  $s_B$  happens with probability  $1 - p$ . Each agent receives income  $e_1$  in period 1 and  $e_2(s)$  in state  $s \in \{s_G, s_B\}$  of period 2, where  $e_2(s_G) \geq e_2(s_B)$ . All households (you can think of them as a single representative household) have the same preferences over consumption

$$(1) \quad U = \frac{c_1^{1-\theta}}{1-\theta} + \beta E \left[ \frac{c_2(s)^{1-\theta}}{1-\theta} \right],$$

where  $\theta \geq 0$ ,  $0 < \beta \leq 1$ , and  $E$  denotes the expectation operator with respect to the state  $s$ . All markets are competitive. Households can buy a bond,  $b$ , at price 1 in period 1 which pays an (endogenous) interest  $1 + r$  in period 2 and is in zero supply. Households start with initial assets of zero, that is they have no bond holdings initially. Note that there is no capital in this economy. There are also no firms (income is obtained by fishing).

### (a) (a) - 5 points

Write down the households state-by-state budget constraints for both periods (you can assume the constraints hold with equality).

Fill in your answer here and/or on sketching paper

---

Maximum marks: 5

### (b) (b) - 5 points

Show that the households' constrained optimization problem is equivalent to maximizing the objective function

$$\tilde{U} = \frac{(e_1 - b)^{1-\theta}}{1-\theta} + \beta E \left[ \frac{(e_2(s) + (1+r)b)^{1-\theta}}{1-\theta} \right],$$

with respect to the bond holdings,  $b$ . (Hint: no proof is required here, just state the procedure of how to derive the above objective function)

Fill in your answer here and/or on sketching paper

---

Maximum marks: 5

### (c) (c) - 10 points

Find the optimality condition with respect to the households' bond holdings and also state the market clearing condition in the bond market. What are the implications of the bond market clearing condition for the equilibrium trading of consumption across time?

Fill in your answer here and/or on sketching paper

---

Maximum marks: 10

(d) **(d) - 5 points**

Consider the bond market clearing condition derived in part (c), what is the consumption of households in period one,  $c_1$ , and in the two states,  $c_2(s_G)$  and  $c_2(s_B)$ , of period 2 then?

Fill in your answer here and/or on sketching paper

Maximum marks: 5

(e) **(e) - 10 points**

Use your results from part (c) to show that in equilibrium the gross interest rate of the bond is given by

(2) 
$$1 + r = \frac{e_1^{-\theta}}{\beta E[e_2(s)^{-\theta}]} = \frac{e_1^{-\theta}}{\beta [pe_2(s_G)^{-\theta} + (1-p)e_2(s_B)^{-\theta}]}.$$

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(f) **(f) - 10 points**

Suppose  $\theta = 1$ . Will the households engage in precautionary savings?

Fill in your answer here and or on sketching paper

Maximum marks: 10

(g) **(g) - 10 points**

Assume now that  $e_1 = 4$  in period 1,  $e_2(s_G) = 6$  and  $e_2(s_B) = 2$  in period 2,  $\beta = 3/4$ ,  $\theta = 1$  and  $p = 1/2$ . What is the gross interest rate  $1 + r$  in equilibrium?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(h) **(h) - 5 points**

Now assume instead that  $e_1 = 4$  in period 1,  $e_2(s_G) = 4$  and  $e_2(s_B) = 4$  in period 2,  $\beta = 3/4$ ,  $\theta = 1$  and  $p = 1/2$ . What is the gross interest rate  $1 + r$  in equilibrium?

Fill in your answer here and/or on sketching paper

Maximum marks: 5

(i) **(i) - 10 points**

Compare the equilibrium interest rate in part (h) to the one in part (g). How do the results relate to precautionary savings?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

3(a) **(a) - 10 points**

What do we mean by Ricardian Equivalence?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

3(b) **(b) - 20 points**

Which three conditions must be fulfilled for Ricardian Equivalence to hold?

Fill in your answer here and/or on sketching paper

Maximum marks: 20

4      **A Three Period Model**

Milton lives for three periods. His income is \$20, 000 in the first period, \$120, 000 in the second period and \$10, 000 in the third period of his life. The interest rate is  $r = 0$  and he has utility function

$$0.5 \log(c_1) + 0.5 \log(c_2) + 0.5 \log(c_3)$$

(a)    **(a) - 5 points**

Find Milton’s optimal consumption choice in the three periods.

Fill in your answer here and/or on sketching paper

Maximum marks: 5

(b)    **(b) - 10 points**

Find the financial asset position at the end of period 1 and 2,  $s_1$  and  $s_2$ , as well as saving  $sav_1, sav_2, sav_3$  in the three periods.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(c)    **(c) - 5 points**

Now suppose that Milton cannot borrow. How do the answers to part (a) and (b) change?

Fill in your answer here and/or on sketching paper

Maximum marks: 5

(d)    **(d) - 5 points**

Suppose the government increases taxes in the first period by \$10, 000 and reduces them in the second period by \$10, 000. How does your answer to question (a) change (if Milton can freely borrow).

Fill in your answer here and/or on sketching paper

Maximum marks: 5

(e) **(e) - 5 points**

Repeat question (d), but now assume Milton cannot borrow.

Fill in your answer here and/or sketching paper

---

Maximum marks: 5