Final Exam II (Solutions)
ECON 4310, Fall 2019

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** subquestion of the long questions.

Good Luck!

	Points	**Max**
Exercise A		**30**
Exercise B		**70**
Exercise C		**30**
Exercise D		**30**
Σ		**160**

Grade: __________

Exercise A (30 Points):

The Solow Model

Consider an economy on its balanced growth path (in steady state) that is characterised by the Solow growth model and by the fact that there is technological progress and no population growth. Now suppose there is a one-time jump in the number of workers.

(a) (10 Points) At the time of the jump, does output per unit of effective labor rise, fall, or stay the same? Why? Explain your answer.

Solution:

Output per unit of effective labor (Y/AL) falls because L increases.

(b) (10 Points) After the initial change (if any) in output per unit of effective labor when the new workers appear, is there any further change in output per unit of effective labor? If so, does it rise or fall? Why? Explain your answer.

Solution:

The economy moves from the steady state to its transition path. Since the Solow growth model exhibits a stable steady state the economy will converge back. In this process output per unit of effective labor will increase.

(c) (10 Points) Once the economy has again reached a balanced growth path, is output per unit of effective labour higher, lower, or the same as it was before the new workers appeared? Why? Explain your answer.

Solution:

Once the economy has again reached a balanced growth path output per unit of effective labour will be the same as prior the shock.

Exercise B (70 Points):

Savings and Uncertainty

Consider a model in which there are two periods ($t = 1, 2$) and a unit mass of identical agents. In period 2 there are two states, denoted by s_G and s_B. The state turns out to be s_G with probability $p \in (0, 1)$ and thus state s_B happens with probability $1 - p$. Each agent receives income e_1 in period 1 and $e_2(s)$ in state $s \in \{s_G, s_B\}$ of period 2, where $e_2(s_G) \geq e_2(s_B)$. All households (you can think of them as a single representative household) have the same preferences over consumption

$$U = \frac{c_1^{1-\theta}}{1-\theta} + \beta \mathrm{E}\left[\frac{c_2(s)^{1-\theta}}{1-\theta}\right], \tag{1}$$

where $\theta \geq 0$, $0 < \beta \leq 1$, and E denotes the expectation operator with respect to the state s. All markets are competitive. Households can buy a bond, b, at price 1 in period 1 which pays an (endogenous) interest $1 + r$ in period 2 and is in zero supply. Households start with initial assets of zero, that is they have no bond holdings initially. Note that there is no capital in this economy. There are also no firms (income is obtained by fishing).

(a) (5 Points) Write down the households state-by-state budget constraints for both periods (you can assume the constraints hold with equality).
Solution:

The budget constraints read

$$c_1 + b = e_1$$
$$c_2(s) = e_2(s) + (1 + r)b, \quad \forall s \in \{s_G, s_B\}.$$

(b) (5 Points) Show that the households' constrained optimization problem is equivalent to maximizing the objective function

$$\widetilde{U} = \frac{(e_1 - b)^{1-\theta}}{1-\theta} + \beta \mathrm{E}\left[\frac{(e_2(s) + (1+r)b)^{1-\theta}}{1-\theta}\right],$$

with respect to the bond holdings, b. (Hint: no proof is required here, just state the procedure of how to derive the above objective function)
Solution:

The households' optimization problem is to maximize U in Equation (1) subject to the period-by-period budget constraints. The objective function $\widetilde{U}$ is derived by simply substituting consumption in U with the period-by-period budget constraints.

(c) (10 Points) Find the optimality condition with respect to the households' bond holdings and also state the market clearing condition in the bond market. What are the implications of the bond market clearing condition for the equilibrium trading of consumption across time?

Solution:

The optimality condition with respect to bond holdings reads (*XX 5 Points XX*)

$$0 = \frac{\partial \widetilde{U}}{\partial b} = -(e_1 - b)^{-\theta} + \beta \mathrm{E}(e_2(s) + (1+r)b)^{-\theta}(1+r). \tag{2}$$

The bond market clearing condition is given by (*XX 3 Points XX*)

$$b = 0,$$

as the bond is in zero supply. Thus, because all household's are identical in equilibrium there will be no trade of consumption across time, the interest rate will adjust such that agents consume exactly their income (*XX 2 Points XX*).

(d) (5 Points) Consider the bond market clearing condition derived in part (c), what is the consumption of households in period one, c_1, and in the two states, $c_2(s_G)$ and $c_2(s_B)$, of period 2 then?

Solution:

Since the bond holdings are zero in equilibrium, the consumption in each state will simply be given by the corresponding income in the particular state

$$\begin{aligned} c_1 &= e_1 \\ c_2(s) &= e_2(s), \quad \forall s \in \{s_G, s_B\}. \end{aligned}$$

(e) (10 Points) Use your results from part (c) to show that in equilibrium the gross interest rate of the bond is given by

$$1 + r = \frac{e_1^{-\theta}}{\beta \mathrm{E}\left[e_2(s)^{-\theta}\right]} = \frac{e_1^{-\theta}}{\beta \left[p e_2(s_G)^{-\theta} + (1-p) e_2(s_B)^{-\theta}\right]}. \tag{3}$$

Solution:

Rewrite the optimality condition in Equation (2) as (writing out the expectation operator)

$$\frac{(e_1 - b)^{-\theta}}{\beta \left[p(e_2(s_G) + (1+r)b)^{-\theta} + (1-p)(e_2(s_B) + (1+r)b)^{-\theta}\right]} = 1 + r,$$

in equilibrium, where $b = 0$, this expression simplifies to

$$\frac{e_1^{-\theta}}{\beta \left[p e_2(s_G)^{-\theta} + (1-p) e_2(s_B)^{-\theta}\right]} = \frac{e_1^{-\theta}}{\beta \mathrm{E}\left[e_2(s)^{-\theta}\right]} = 1 + r.$$

(f) (10 Points) Suppose $\theta = 1$. Will the households engage in precautionary savings?

Solution:

We have precautionary savings when the marginal utility is strictly convex or the third derivative of the utility function is positive. Here:

$$\begin{aligned}
U'(c) &= c^{-\theta} \\
U''(c) &= -(\theta) * c^{-(1+\theta)} \\
U'''(c) &= (\theta)(1+\theta) * c^{-(2+\theta)} > 0 \quad for \quad \theta = 1
\end{aligned}$$

(g) (10 Points) Assume now that $e_1 = 4$ in period 1, $e_2(s_G) = 6$ and $e_2(s_B) = 2$ in period 2, $\beta = 3/4$, $\theta = 1$ and $p = 1/2$. What is the gross interest rate $1 + r$ in equilibrium?

Solution:

The equilibrium gross interest rate is given by

$$1 + r = \frac{1/4}{3/4\,[1/2 \times 1/6 + 1/2 \times 1/2]} = \frac{1}{3 \times (1/12 + 1/4)} = 1.$$

(h) (5 Points) Now assume instead that $e_1 = 4$ in period 1, $e_2(s_G) = 4$ and $e_2(s_B) = 4$ in period 2, $\beta = 3/4$, $\theta = 1$ and $p = 1/2$. What is the gross interest rate $1 + r$ in equilibrium?

Solution:

The equilibrium gross interest rate is given by

$$1 + r = \frac{1/4}{3/4\,[1/2 \times 1/4 + 1/2 \times 1/4]} = \frac{1}{3 \times (1/8 + 1/8)} = 4/3 > 1.$$

(i) (10 Points) Compare the equilibrium interest rate in part (h) to the one in part (g). How do the results relate to precautionary savings?

Solution:

On average, the second period income under both scenarios is equal to the first period income. However, the scenario in part (g) features risk while there is no risk in part (h) concerning the second period income, and the interest rate is falling in the amount of risk (*XX 5 Points XX*). Since the marginal utility of the stated preferences is strictly convex, there is a precautionary savings motive for the households which is reflected by the fact that in the economy with risk (part (g)) the equilibrium interest rate is smaller than in the economy without risk, because the interest rate has to fall to accommodate the precautionary savings motive and to bring the bond market into the zero bond demand equilibrium (*XX 5 Points XX*).

Exercise C (30 Points):

Ricardian Equivalence

(a) (10 Points) What do we mean by Ricardian Equivalence?

Solution:

The timing of taxes to finance government spending is irrelevant. Households will not change their decisions when the timing of taxes change as long their intertemporal budget constraint is unchanged.

(b) (20 Points) Which three conditions must be fulfilled for Ricardian Equivalence to hold?

Solution:

1.(6 points) Households are infinitely lived or equivalently there are dynasties who are altruistic towards their children. 2. (7 points) Households can freely borrow and save (no borrowing limits). 3. (7 points) There are lumpsum taxes (taxes on labor, capital and consumption are distortive)

Exercise D (60 Points):

A Three Period Model

Milton lives for three periods. His income is \$20,000 in the first period, \$120,000 in the second period and \$10,000 in the third period of his life. The interest rate is $r = 0$ and he has utility function

$$0.5\log(c_1) + 0.5\log(c_2) + 0.5\log(c_3)$$

(a) (5 Points) Find Milton's optimal consumption choice in the three periods.

Solution:

Since $\beta(1+r) = 1$, $c_1 = c_2 = c_3 = 50,000$

(b) (10 Points) Find the financial asset position at the end of period 1 and 2, s_1 and s_2, as well as saving sav_1, sav_2, sav_3 in the three periods.

Solution:

$$\begin{aligned}
sav_1 &= 20,000 - 50,000 = -30,000 \\
s_1 &= s_0 + sav_1 = -30,000 \\
sav_2 &= 120,000 - 50,000 = 70,000 \\
s_2 &= s_1 + sav_2 = 40,000 \\
sav_3 &= 10,000 - 50,000 = -40,000 \\
s_3 &= s_2 + sav_3 = 0
\end{aligned}$$

(c) (5 Points) Now suppose that Milton cannot borrow. How do the answers to part (a) and (b) change?

Solution:

Milton had initially wanted to borrow in period 1, but can no longer do so. From period 2 onwards he is not borrowing constrained, since he planned to save anyway. He therefore chooses smooth consumption over the last two periods of his life.

$$c_1 = 20,000 \qquad c_2 = 65,000 \qquad c_3 = 65,000$$

(d) (5 Points) Suppose the government increases taxes in the first period by \$10,000 and reduces them in the second period by \$10,000. How does your answer to question (a) change (if Milton can freely borrow).

Solution:

Since Milton can freely borrow, Ricardian Equivalence applies and the tax policy has no effect.

(e) (5 Points) Repeat question (d), but now assume Milton cannot borrow.

Solution:

Milton continues to be borrowing constrained in the first period. In fact, the borrowing constraint is exacerbated by the tax policy. His consumption is given by

$$c_1 = 10,000 \qquad c_2 = 70,000 \qquad c_3 = 70,000$$