Final Exam Retake

ECON 4310, Fall 2020

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	Max
Exercise A		**40**
Exercise B		**40**
Exercise C		**60**
Σ		**140**

Grade: __________

Exercise A:

Taxation and Labor Supply (40 points)

Consider a household that chooses consumption c and labor supply l to solve the following maximization problem

$$\max_{c,l\geq 0} u\left(c,l\right)$$
$$\text{s.t.}$$
$$c=(1-\tau)l,$$

where τ is the labor income tax rate. Note that the wage per hour work is set to one.

(a) (10 Points) Consider the following utility function $u\left(c,l\right)=\frac{c^{1-\sigma}}{1-\sigma}-\frac{l^{1+\sigma}}{1+\sigma}$. Derive the optimality condition, relating labor supply and consumption. Solve for labor supply l, as a function of τ.
ANS:

$$\begin{aligned}
\left(1-\tau\right)c^{-\sigma} &= l^{\sigma} \Rightarrow \\
l &= \left(1-\tau\right)^{\frac{1}{\sigma}} c^{-1} \\
&= \left(1-\tau\right)^{\frac{1}{\sigma}} \left((1-\tau)l\right)^{-1} \\
l^2 &= \left(1-\tau\right)^{\frac{1}{\sigma}-1} \Rightarrow \\
l &= \left(1-\tau\right)^{\frac{1-\sigma}{2\sigma}}.
\end{aligned}$$

(b) (10 Points) Suppose that $\sigma=2$. How does labor respond to labor tax increase? Explain your answer by analyzing the income and the substitution effects of labor supply.

(c) (10 Points) Now consider an alternative utility function $u\left(c,l\right)=\log\left(c-\frac{l^{1+\sigma}}{1+\sigma}\right)$. Derive the optimality condition, relating labor supply and consumption. Solve for labor supply l.
ANS:

$$\begin{aligned}
1-\tau &= l^{\sigma} \Rightarrow \\
l &= \left(1-\tau\right)^{\frac{1}{\sigma}}.
\end{aligned}$$

(d) (10 Points) Suppose that $\sigma=0.5$. How does labor respond to a labor tax increase with the alternative utility function? Explain your answer by analyzing the income and the substitution effects of labor supply.

Exercise B:

Ricardian Equivalence (40 Points)

Consider an 2-period OLG model where one period is 30 years and population growth is 0. Suppose that at the beginning of period t, the government announces a special new one-time expenditure plan: spending g per capita in that period to reduce the global warming. The government has come up with three policies to finance the expenditure:

1. Finance the expenditure with immediate lumpsum taxes on all households living in period t.
2. Issue government debt, and repay that debt, including interest, in the next period using lumpsum taxes on all households living in period $t+1$.
3. Issue government debt and simply pay the interest on that government debt forever, without ever redeeming the debt itself.

Assume that the interest rate is $r = 100\%$ (so that $1 + r = 2$).

Now consider a household who lives in period t and $t+1$, i.e., she is young in period t and old in period $t+1$. She works when she is young and old. Her labor incomes are the same when she is young and when she is old: $w_t^y = w_{t+1}^o$. She has utility function

$$\log(c_t^y) + \beta \log(c_{t+1}^o),$$

where $\beta = 1/2$, and c_t^y and c_{t+1}^o denote her consumption in period t when she is young and consumption in period $t+1$ when she is old, respectively.

(a) (10 Points) Assume that the household can borrow and lend as much as she wants. How does she rank those three tax policies? Explain.

(b) (10 points) Now suppose that the household can lend but cannot borrow any longer and $g = w_t^y/4$. How does she rank those three tax policies? Explain.
ANS: 3>2>1, because in 1, borrowing constraint is binding.

(c) (10 points) Now consider a household from two generates later, who lives in period $t+2$ and $t+3$, and she can borrow and lend freely. How does she rank those tax policies? Explain.

(d) (10 points) Finally, suppose that all households are the same and can live forever instead of just 2 periods. Furthermore, assume that households work and receive the same labor income every period, i.e., $w_t = w$. How does the representative household rank those tax policies and how does her ranking depend on how much she can borrow? Explain. (Hint: you just need to generally and qualitatively discuss the relation between the ranking and the borrowing constraint and you don't need to solve for any expression or number.)

Exercise C:

A Real Business Cycle Model (60 Points)

Consider a representative household of a closed economy. The household has a planning horizon of two periods and is endowed with the following preferences over consumption, c,

$$U = u(c_1) + \beta \mathrm{E} u(c_2(s_2)),$$

where $\beta = 1/3$, and $u(c) = \log c$ (natural log utility function).

The variable s_2 denotes the state of the economy in the second period which follows the stochastic process

$$s_2 = \begin{cases} s_G, \text{ with prob. } p \\ s_B, \text{ with prob. } 1-p, \end{cases}$$

where $p = 0.5$ and the household conditions the consumption, $c_2(s_2)$, in the second period on the state, s_2. Assume the household's labor supply is exogenous and always equal to 1.

Labor market assumptions:

Assume that in each period and in each state of the economy, s_t, there is a linear (in labor n_t) production technology of the form

$$y_t(s_t) = A_t(s_t) n_t(s_t),$$

and the labor market is assumed to be competitive. Assume the labor productivity in the first period is given by $A_1 = A = 1$, and the labor productivity is higher in the good state of the second period,

$$A_2(s_G) = 1 + \epsilon > A_2(s_B) = 1 - \epsilon, \quad \epsilon = 1$$

than in the bad state of the second period. The wages are denoted as w_1, $w_2(s_G)$, and $w_2(s_B)$.

Asset market assumptions:

Assume the household does have access to a risk-free asset, a_2, and the associated interest rate is denoted as $r_2 = 200\%$.

We can denote a variable x in period 2 in stage G or B simply as x_{2G}. For example, $A_2(s_G)$ as A_{2G}, $w_2(s_B)$ as w_{2B}, $c_2\,(s_G)$ as c_{2G} and $\lambda_2\,(s_G)$ as λ_{2G}, etc.

(a) (5 Points) Write down the state-by-state budget constraints for the household.

(b) (5 Points) Let $(\lambda_1, \lambda_{2G}, \lambda_{2B})$ denote the Lagrange multipliers of the state-by-state budget constraints. State the representative agent's Lagrangian. (Note that the expected utility for the second period is the summation of utility across good and bad states, weighted by probability, i.e., $\mathrm{E}u(c_2) = pu(c_{2G}) + (1-p)u(c_{2B})$. The values of the Lagrange multipliers are not known yet, so you don't need to find their values at this stage)

(c) (10 Points) Derive the optimality conditions with respect to consumption and savings, i.e., $c_1, c_2(s_G), c_2(s_B)$ and a_2 by using multipliers.

(d) (10 Points) Derive the stochastic consumption Euler equation (only involves with c_1,c_{2G},c_{2B}, β and r_2 and no multipliers).

(e) (10 Points) Solve for optimal consumption, savings. (Now you need to find the numerical values for c_1, c_{2G}, c_{2B} and a_2. Hint: the Euler equation can be transformed into a quadratic equation of a_2, and the positive root is the optimal a_2).
ANS:

$$
\begin{aligned}
\frac{1}{1-a} &= \beta(1+r_2)\left(p\frac{1}{(1+r_2)a+1+\epsilon} + (1-p)\,\frac{1}{(1+r_2)a+1-\epsilon}\right) \Rightarrow \\
\frac{1}{1-a} &= \left(\frac{1}{2}\frac{1}{3a+1+\epsilon} + \frac{1}{2}\frac{1}{3a+1-\epsilon}\right) \Rightarrow \\
\frac{1}{1-a} &= \frac{(3a+1)}{(3a+1+\epsilon)\,(3a+1-\epsilon)} \Rightarrow \\
(3a+1)^2 - \epsilon^2 &= (3a+1)\,(1-a) \Rightarrow \\
(3a+1)\,(3a+1-1+a) - \epsilon^2 &= 0 \Rightarrow \\
(3a+1)\,4a - \epsilon^2 &= 0 \Rightarrow \\
\\
12a^2 + 4a - \epsilon^2 &= 0 \Rightarrow \\
a &= \frac{-4 \pm \sqrt{16+48\epsilon^2}}{24} \\
&= \frac{-4 \pm 4\sqrt{1+3\epsilon^2}}{12} \\
&= \frac{-1 \pm 2}{3}
\end{aligned}
$$

(f) (10 Points) If the variance of income shocks becomes larger, i.e., ϵ becomes larger, what happens to the saving? Why?

(g) (10 Points) Is there an interest rate for the risk-free asset that makes the household optimally choose not to save? Why?