

**Exercise A:****A Simple Life-cycle Model: Warming up (20 points)**

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Sunny and Cher both obey a two period life-cycle model of consumption. Both starts out with \$100, which they have inherited from their Grandma. Sunny earns \$100 in the first period, while Cher earns \$50. In the second period Cher, who is a college graduate, earns 3 times more than Sunny. Both can borrow and lend at interest rate  $r$ . You observe that Sunny consumes \$150 in both periods while Cher consumes \$200 in the first period and \$230 in the second period. What is the interest rate  $r$ ?

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**Exercise B:****A Simple Life-cycle Model (100 points)**

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Joe Six-pack just learned about our two period life cycle model and fell so deeply in love with it that he decides that he wants to live (i.e. consume) according to it. He is in the first period of his life and he spends it in college. In college he works in the cafeteria where he makes \$20,000 after taxes. He knows for sure that he is going to get a job at McKinsey (he is an economics major) and will make \$110,000 after taxes in the second period of his life (sadly enough he only lives for two periods). He has no initial wealth and he can borrow and lend at an interest rate of 10%. His utility function is

$$U(c_1, c_2) = 2\sqrt{c_1} + \sqrt{c_2}$$

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- (a) (10 Points) Determine Joe's optimal consumption in both periods of his life. Is Joe a saver or a borrower? Determine his optimal level of saving/borrowing.
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- (b) (10 Points) How would Joe's optimal consumption and saving decision change if he had a utility function of the form  $U(c_1, c_2) = \min\{c_1, c_2\}$ ? This is a so called Leontief utility function.
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- (c) (20 Points) Back to Joe's original utility function for the rest of the question. Suppose the interest rate increases to 16%. Now what is Joe's optimal consumption and saving/borrowing plan? Discuss the change in  $c_1$  and  $c_2$  in light of the income, substitution and human wealth effects.
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- (d) (10 Points) Is Joe better off or worse off after the interest rate increase? Justify your answer and provide some economic intuition.
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- (e) (10 Points) Suppose the interest rate is back at 10%. The government wants to cut taxes to stimulate the economy. Suppose the government cuts the taxes that Joe has to pay in the first period by \$2,000, so that Joe's after tax income increases from \$20,000 to \$22,000. In order to finance this tax cut the government has to increase taxes in the second period by  $\$2000 * (1 + r) = \$2,200$ . Hence Joe's income in the second period goes down from \$110,000 to \$107,800. What is Joe's new optimal consumption and saving plan. Compare to your answer in Part (a).
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- (f) (10 Points) Now suppose that Joe is borrowing constrained, in particular assume that he can't borrow at all in the first period of his life. What is the optimal consumption plan now (incomes in both periods are at their original levels)?
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- (g) (10 Points) Joe is still borrowing constrained, and now the government carries out the same tax change as in (e). Compute Joe's new optimal consumption plan and compare to the results in the previous question. Explain.
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- (h) (20 Points) Suppose more generally that Joe's utility is

$$U(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \beta \frac{c_2^{1-\sigma}}{1-\sigma}$$

$\sigma > 0$ . He has no initial savings, income  $y_1$  in period 1 and  $y_2$  in period 2. The interest rate is  $r$ . What can you say about the effect of a change in the interest rate on savings,  $\frac{ds}{dr}$ , and consumption in the first period,  $\frac{dc_1}{dr}$ ? How does this depend on the parameter  $\sigma$ ? Relate your answer to the income, substitution and human wealth effects.

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