## Solution Proposal

## Part A

Here are criteria for answering part A well.

1. The student should demonstrate an understanding of the main criteria of a "good" model in (positive) economics in Friedman (1953).
2. The student should demonstrate an understanding what the Lucas critique is and what Lucas' criteria of a "good" model is (Lucas, 1976).
3. The student should demonstrate an understanding of Gerdrup and Nicolaysen (2011).
(a) The student should provide a brief explanation of what a medium-sized DSGE model is (e.g., NEMO) and explain how it satisfies/does not satisfy the criteria in Friedman (1953) and Lucas (1976).
(b) The student should provide a brief explanation of what SAM is and explain how it satisfies/does not satisfy the criteria in Friedman (1953) and Lucas (1976).
4. The essay should be well-structured and well-written.

If all four criteria are satisfied, the student should get a full score.

## Part B

1. Example solution by Lagrange:

$$
\mathcal{L}=\frac{c_{1}^{1-\sigma}}{1-\sigma}+\frac{c_{2}^{1-\sigma}}{1-\sigma}-\lambda_{1}\left(c_{1}+b_{1}-y\right)-\lambda_{2}\left(c_{2}-(1+r) b_{1}\right)
$$

FOCs:

$$
\begin{aligned}
& \frac{\partial \mathcal{L}}{\partial c_{1}}=c_{1}^{-\sigma}-\lambda_{1}=0 \\
& \frac{\partial \mathcal{L}}{\partial c_{2}}=c_{2}^{-\sigma}-\lambda_{2}=0 \\
& \frac{\partial \mathcal{L}}{\partial b_{1}}=-\lambda_{1}+(1+r) \lambda_{2}=0
\end{aligned}
$$

which together give

$$
c_{1}^{-\sigma}=(1+r) c_{2}^{-\sigma} .
$$

2. The interest rate is the price of goods in period 1 relative to period 2. If the interest rate is above 0 in this case, consumption is relatively cheaper in period 2 and the household will move resources to period 2 by saving in period 1 and consuming later. Consumption growth, $c_{1} / c_{2}$ is therefore increasing in the interest rate.
3. The euler equation implies that

$$
c_{2}=(1+r)^{1 / \sigma} c_{1} .
$$

Combine the two budget constraints to get

$$
c_{1}+\frac{c_{2}}{1+r}=y .
$$

Combining the euler and the budget constraints, we get

$$
c_{1}+(1+r)^{1 / \sigma-1} c_{1}=y
$$

which becomes

$$
c_{1}=\frac{y}{1+(1+r)^{1 / \sigma-1}} .
$$

4. The effect of an interest rate change on the $c_{1}$ is ambiguous. There are two effects, an income and a substitution effect. A higher interest rate raises the price of con-
sumption in period 1 relative to period 2 , resulting in a reduction in consumption in period 1. The income effect, on the other hand, works in the opposite direction. Because the household has income only in period 1, it is originally planning to save. A higher interest rate makes its consumption plan cheaper, and the household is therefore richer, resulting in higher consumption in period 1 and 2.
$1 / \sigma$ governs the strength of the substitution effect, while 1 is the strength of the income effect. If $1 / \sigma>1$, the substitution effect dominates and the household reduces consumption in period 1.
5. The euler equation in the modified solution implies that

$$
c_{1}^{-\sigma}=c_{2}^{-\gamma} .
$$

One way to illustrate that $c_{1}>c_{2}$ if $\sigma>\gamma$ is to consider the case where $c_{1}=c_{2}$, then the Euler does not hold because the marginal utility in period 1 is higher than the marginal utility in period 2. In this case, the household would gain from moving resources from period 2 to period 1. Hence, $c_{1}$ must be greater than $c_{2}$ if $\sigma>\gamma$ in optimum. The student can use other methods to illustrate, including verbal arguments.

## Part C

1. The law of motion for capital in the Solow growth model is derived using the capital accumulation formula where capital next period is equal to investment in the current period plus the remaining capital after depreciation:

$$
K_{t+1}=I_{t}+(1-\delta) K_{t}
$$

and replace for $I_{t}$

$$
K_{t+1}=s Y_{t}+(1-\delta) K_{t}
$$

which is equal to

$$
K_{t+1}=s F\left(K_{t}, L\right)+(1-\delta) K_{t} .
$$

2. In the steady state, we have that $K_{t+1}=K_{t}=K_{s s}$ and $Y_{s s}=F\left(K_{s s}, L\right)$, which implies
that the law of motion of capital becomes

$$
\begin{aligned}
K_{s s} & =s Y_{s s}+(1-\delta) K_{s} s \\
\delta K_{s s} & =s Y_{s s} \\
K_{s s} / Y_{s s} & =s / \delta
\end{aligned}
$$

3. Using the Cobb-Douglas production function with $L=1$, we get

$$
s K_{s s}^{\alpha}=\delta K_{s s}
$$

which implies that

$$
K_{s s}=\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}
$$

and thus that

$$
Y_{s s}=K_{s s}^{\alpha}=\left(\frac{S}{\delta}\right)^{\frac{\alpha}{1-\alpha}}
$$

4. We know that

$$
Y_{s s}=\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}
$$

taking the derivative with respect to the saving rate, we get

$$
\frac{\partial Y_{s s}}{\partial s}=\frac{\alpha}{1-\alpha}\left(\frac{s}{\delta}\right)^{-\frac{1}{1-\alpha}} \frac{1}{\delta}=\frac{\alpha}{1-\alpha}\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{s}
$$

which is always positive.
5. Steady state consumption is

$$
C_{s s}=(1-s) Y_{s s}
$$

and the derivative with respect to the $s$ is

$$
\frac{\partial C_{s s}}{\partial s}=\frac{\alpha}{1-\alpha} \frac{1-s}{s} Y_{s s}-Y_{s s}
$$

The first term is the effect that a higher saving rate has on higher output and thus consumption, and the second term is the direct effect of consuming less due to a higher saving rate.

