International Risk-sharing

ECON4330 Lecture 3 Spring 2014 Revised

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Relation to text

OR ch 1: Global Two countries, two periods, no risk
OR Ch 2.3: One country, infinite horizon, risk, no sharing
OR Ch. 3: Two countries, two periods, risk sharing

Emphasis: 5.1.1-5.1.6, 5.2.1, 5.3.3 and Boxes 5.1-5.3

What would a fully integrated world economy look like?
How integrated are the world capital markets actually?

Assumptions

- Single commodity
- Endowment economy
- Competitive equilibrium
- Two periods
- Two states of nature
- Two countries

Arrow-Debreu Markets

- Commodities distinguished by time, t, and state of nature, s
- All trade at beginning of period 1, with period 2's state unknown

q_{2s}units of the commodity in period 1 buys one unit in period 2 if state s occurs
q_{2s} is the time 1 price of a contingent claim
Budget constraint for a home consumer is

$$C_1 + q_{21}C_{21} + q_{22}C_{22} = Y_1 + q_{21}Y_{21} + q_{22}Y_{22} \tag{1}$$

$Y_{ts}, \ C_{ts}$ Output and consumption at time t and in state s.

The safe asset

Buying one unit of the commodity in every state costs $q_{21} + q_{22}$.

This creates a safe asset

The time 1 price of a safe asset has earlier been denoted $1/(1+r)$. Hence,

$$\frac{1}{1+r} = q_{21} + q_{22} \Longleftrightarrow 1 + r = 1/(q_{21} + q_{22})$$

Define period 2 prices

$$p_{21} = q_{21}(1+r), \;\; p_{22} = q_{22}(1+r) \tag{2}$$

$p_{t,s} =$ is the price of a claim contingent on state s relative to the price on an unconditional claim

$$p_{2s} = q_{2s/[1/(1+r)]}$$

$$p_{21} + p_{22} = 1 \tag{3}$$

Buying one unit in each state is the same as buying the safe asset

Actuarial fairness

π_s =probability that state s occurs

- if you buy one unit conditional on that state s occurs, the expected value of what you get in period 2 is π_s
- prices are said to be actuarially fair when what you pay is equal to the expected value of what you get, here when $p_{ts} = \pi_s$
- even if the market is competitive, prices will not always be fair

Budget

In terms of safe interest rate and period 2 prices :

$$C_1 + (1+r)^{-1}(p_{21}C_{21} + p_{22}C_{22}) = Y_1 + (1+r)^{-1}(p_{21}Y_{21} + p_{22}Y_{22}) \quad (4)$$

Net purchases of contingent claims are: $B_{ts} = C_{ts} - Y_{ts}$

Then from reorganizing (4)

$$\frac{p_{21}}{1+r}B_{21} + \frac{p_{22}}{1+r}B_{22} = Y_1 - C_1 \quad (5)$$

Left: Net lending, Right, Current account surplus

Choice

Max expected utility:

$$U = u(C_1) + \pi_1 \beta u(C_{21}) + \pi_2 \beta u(C_{22}) \tag{6}$$

Budget

$$C_1 + (1+r)^{-1}(p_{21} C_{21} + p_{22} C_{22}) = Y_1 + (1+r)^{-1}(p_{21} Y_{21} + p_{22} Y_{22}) = W$$

W = total wealth

First order conditions :

$$p_{2s} u'(C_1) = \beta(1+r)\pi_s u'(C_{2s}), \; s = 1, 2 \tag{7}$$

- Standard consumption Euler equation if prices are actuarially fair

First order conditions

$$p_{2s} u'(C_1) = \beta(1+r)\pi_s u'(C_{2s}), \; s = 1, 2 \;\; (7)$$

From (7) by adding over s:

$$u'(C_1) = \beta(1+r)[\pi_1 u'(C_{21}) + \pi_2 u'(C_{22})] = \beta(1+r)\mathbb{E}u'(C_{2s}) \quad (8)$$

From (7) by dividing one equation by the other

$$\frac{\pi_2 u'(C_{22})}{\pi_1 u'(C_{21})} = \frac{p_{22}}{p_{21}} \quad (9)$$

Marginal rate of substitution = relative price
Full insurance ($C_{21} = C_{22}$) only if $p_{22}/\pi_2 = p_{21}/\pi_1$. Actuarially fair premia.

CRRA Utility

$u(c) = [1/(1-\theta)]c^{1-\theta}$Utility function
$u'(c) = c^{-\theta} > 0$ Non-satiation
$u''(c) = -\theta c^{-\theta-1} < 0$ Risk aversion
$u'''(c) = \theta(1+\theta)c^{-\theta-2} > 0$ Prudence

$\theta=$ degree of relative risk aversion$= 1/\sigma$, $\sigma=$ intertemporal elasticity of substitution
Smoothing consumption over time and over states

CRRA Continued

First order conditions with CRRA (compare (7))

$$\frac{p_{2s}}{1+r}C_1^{-\theta} = \pi_s \beta C_{2s}^{-\theta} \Longleftrightarrow C_{2s} = [\beta(1+r)\pi_s/p_{2s}]^{1/\theta}C_1 \tag{10}$$

Consumption growth rates $C_{21}/C_1$and C_{22}/C_1 the same for everyone with

the same $\beta$and θ.

- Use (10) to eliminate $C_{21}$and $C_{22}$from budget equation, then solve for C_1
- Consumption levels proportional to the country's total wealth W.

Equilibrium conditions

$$C_1 + C_1^* = Y_1 + Y_1^* = Y_1^W \tag{11}$$

$$C_{2s} + C_{2s}^* = Y_{2s} + Y_{2s}^* = Y_{2s}^W \; s = 1, 2 \tag{12}$$

Global output Y^W.
Since everyone has the same consumption growth rates, these have to be equal to the growth rate of world output:

$$\frac{C_{2s}^*}{C_1^*} = \frac{C_{2s}}{C_1} = \frac{Y_{ts}^W}{Y_1^W}$$

Equilibrium prices

To get period 2 prices for contingent claims:Insert for Y_{2s}^W/Y_1^W for C_{2s}/C_1 in first order conditions (10)

$$p_{2s} = \pi_s \beta(1+r)\left[\frac{Y_{2s}^W}{Y_1^W}\right]^{-\theta} \tag{13}$$

Surprise! Higher output means lower relative price
Sum prices to get reduced form for r:

$$\sum p_{2s} = \beta(1+r)(Y_1^W)^{\theta} \sum \pi_s \left[Y_{2s}^W\right]^{-\theta} = 1$$

$$1+r = \frac{(Y_1^W)^{-\sigma}}{\beta \sum_{s=1}^{\mathcal{S}} \pi_s \left[Y_{2s}^W\right]^{-\theta}} \tag{14}$$

From (13) and (14)

$$p_{2s} = \frac{\pi_s \left[Y_{2s}^W\right]^{-\theta}}{\sum_{j=1}^{\mathcal{S}} \pi_j \left[Y_{2j}^W\right]^{-\theta}} \tag{15}$$

Risk sharing

- Consumption growth the same in all countries, even if income growth differs
- Consumption growth the same in all states that have the same world output
- Full insurance against macro risk impossible
- Full insurance and acturarially fair prices, $p_{2s} = \pi_s$, only if world output is the same in all states
- then 1.order conditions are reduced to

$$C_{2s} = [\beta(1+r)]^{+\sigma} C_1$$

Same consumption in all states in period 2

Unfair prices?

Insuring against the state with lowest world output costs more
Two-state example
From (15)

$$p_{22} = \frac{\pi_2 \left[Y_{22}^W\right]^{-\theta}}{\pi_1 \left[Y_{21}^W\right]^{-\theta} + \pi_2 \left[Y_{22}^W\right]^{-\theta}} = \frac{\pi_2}{\pi_1 \left[Y_{22}^W / Y_{21}^W\right]^{\theta} + \pi_2}$$

If $Y_{22}^W < Y_{21}^W$, denominator to the right is less than one and $p_{22} > \pi_2$.

- If insurance for macro risk were sold at a "fair" price, there would be only buyers, no sellers.

Precautionary saving

- Uninsurable macro risk increases the incentive to save for CRRA-consumers
- Global savings are zero anyway
- Equilibrium effect is reduced interest rate

From (10)

$$1 + r = \frac{(1/\beta)(Y_1^W)^{-\theta\sigma}}{\pi_1 \left[Y_{21}^W\right]^{-\theta} + \pi_2 \left[Y_{22}^W\right]^{-\theta}}$$

With certainty and same expectation:

$$1 + r = \frac{(1/\beta)(Y_1^W)^{-\sigma}}{[\pi_1 Y_{21}^W + \pi_2 Y_{22}^W]^{-\sigma}}$$

The convexity of marginal utility ensures that the latter is bigger than the former

Extensions

- Arrow-Debreu securities can be replaced by other securities
- Need at least as many as there are states of nature
- More periods: Sequential trade can give same result
- Capital can be included
- Separation of investment decision and savings decision

Three Puzzles

- Low consumption correlation (Backhus, Kehoe, Kydland)
- High correlation between saving and investment (Feldstein-Horioka)
- Home bias in portfolios of marketed securities

Correlations

	A	B
US	1.00	0.63
Australia	-0.09	0.44
Canada	0.53	0,67
France	0.37	0.58
Germany	0.37	0.44
Italy	0.01	0.80
Japan	0.35	0.47

Column A: own with US consumption Column B: investment and saving. After detrending Source: Palgraves Dictionary of Economics.

Possible explanations

- Incomplete markets, uninsurable risks
- Keynesian mechanisms
- Non-traded goods

Incomplete markets

- Shiller: Missing macro markets
- Uninsurable risks