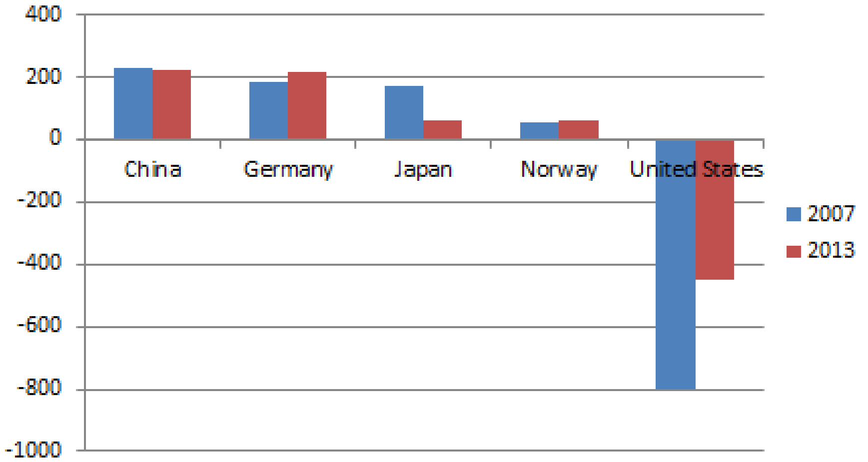
The current account in an intertemporal equilibrium model

Econ 4330 International Macroeconomics Spring 2014

First lecture

Asbjørn Rødseth January 14 2014

Current account surpluses. Billion USD

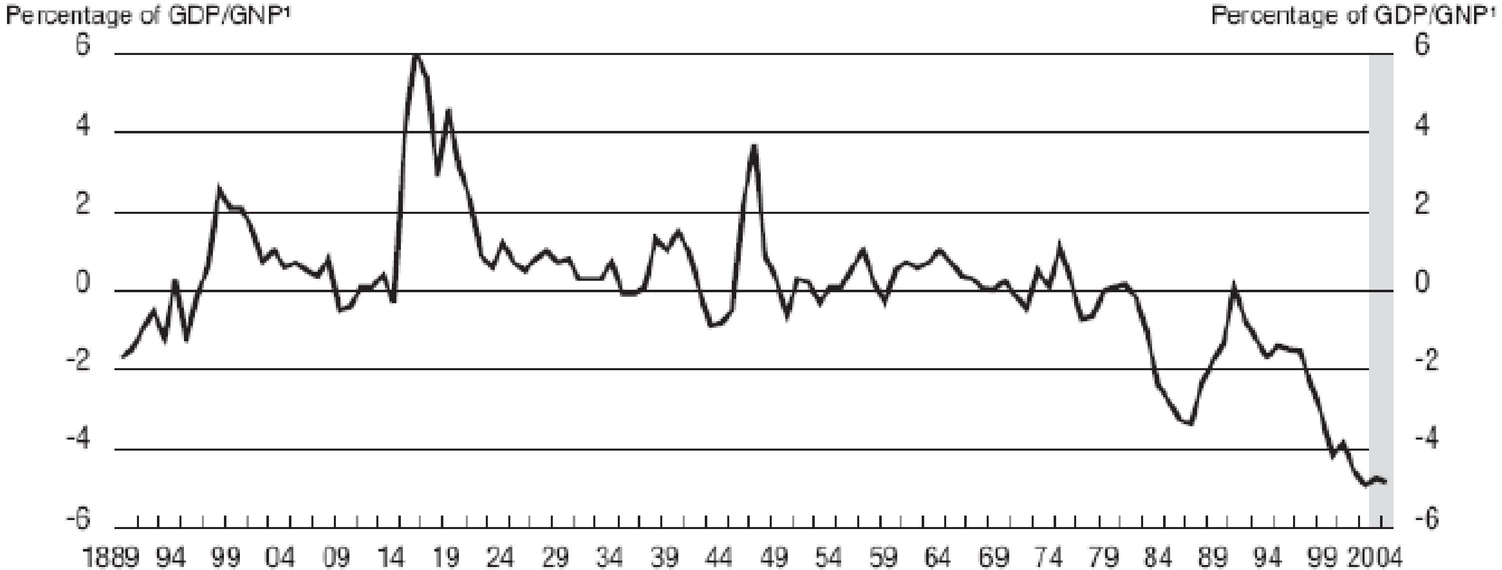


Country	Per cent of GDP 2006	2012
Kuwait	43,0	43
Nigeria	12	8
Norway	16	14
Saudi Arabia	27	23
Venezuela	15	3

Country	Per cent of GDP
Sao Tomé	- 45.9
Iceland	- 27.3
Latvia	- 21.1
Estonia	- 15.5
Greece	- 9.6
Portugal	- 9.4
Spain	- 8.6

Selected deficit countries 2006

Figure 1. **The US current account in historical perspective**
Percentage of GDP/GNP[1]



1. GNP before 1929.

Source: OECD, US Bureau of Economic Analysis; and for the pre-1946 period Bureau of the Census: Historical Statistics of the United States, Washington DC, 1975.

Questions

- What determines current account deficits and surpluses?
- How are they affected by fiscal and monetary policy?
- Can deficits be sustained? For how long?
- Will they self-correct or do they warrant policy changes?
- How does the current account behave during business cycles?
- Is a current account deficit a threat to employment?
- Can a current account deficit force a country to devalue?

Approaches

The intertemporal approach

Intertemporal general equilibrium models, explicit optimization over time, no nominal rigidities. Representative consumers and producers. Countries treated as if they were individuals. *Obstfeld and Rogoff.*

The traditional macro approach

Less focus on explicit optimization in micro, more focus on macro behavioral equations that seem to have empirical support. Nominal rigidities and unemployment problems. *Rødseth.*

Current account - definition

Current account =

+ Trade account

  Exports minus imports of goods and services

+ Primary income account

  Payments for the use of labor and financial resources

+ Secondary income account

  Redistribution (foreign aid, remittances etc)

The accumulation equation

Net foreign assets at the beginning of the period

+ Current account surplus $\leftarrow$ Transactions

+ Revaluations

= Net foreign assets at the end of the period

Current account surplus = Net investment in foreign assets

Relation to investment and saving

Saving = Net investment in real capital

+ Net investment in financial assets

= Investment in real capital at home

+ Current account surplus

Current account surplus = Saving – Net investment in real capital

Current account, saving and investment in real capital in per cent of GDP 2006

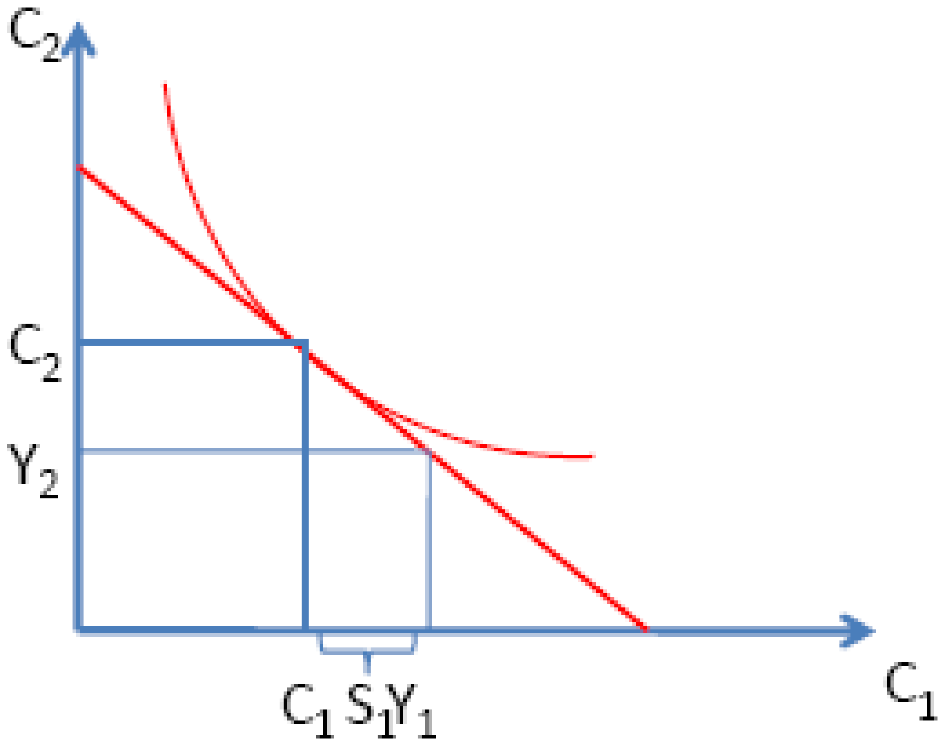
Country	Current account	Saving	Investment	Gov. Surplus
Germany	5,0	22,8	17,8	-1,6
Japan	3,9	28,0	24,1	-4,1
United Kingdom	-3,2	14,8	18,0	-2,7
United States	-6,2	14,1	20,0	-2,6
World		23,3	23,0	

The figures are for *gross* saving and *gross* investment

The simplest possible model

- The economy exists for two periods, labeled 1 and 2
- Small open economy. Everyone can borrow and lend at a given world market interest rate, $\boldsymbol{r}$
- One good at each date, consumed in quantities C_1 and C_2
- Endowment economy: Output in each period is given: Y_1 and Y_2
- Representative consumer: All individuals are identical, population size normalized to one.
- Perfect foresight (no uncertainty)

The model of consumer saving from ECON1210 turned into a model of the current account of an entire country.



C2
C2
Y2
C1 S1Y1
C1

Period budget constraints (B_2=net lending to abroad)

$$C_1 + B_2 = Y_1 \quad (1)$$

$$C_2 = Y_2 + (1+r)B_2 \quad (2)$$

The current account is by definition

$$CA_1 = S_1 = Y_1 - C_1 = B_2$$

$$CA_2 = S_2 = Y_2 + rB_2 - C_2 = -B_2 = -CA_1$$

The present-value budget constraint (from (1) and (2))

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \qquad (3)$$

[Hint: Solve (2) for B, insert in (1).]

Consumer maximizes

$$U = u(C_1) + \beta u(C_2) \quad (4)$$

Subject to present-value budget constraint

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \qquad (3)$$

$\beta = 1/(1+\delta)$, $0 < \beta < 1$, subjective discount *factor,* $\delta > 0$*,* discount *rate*

Assumptions

$$u'(C) > 0, \;\; u''(C) < 0, \qquad \lim_{C \to 0} u'(C) = \infty$$

First order condition

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r} \quad (4)$$

MRS = Price ratio

1/(1+r) = price of consumption in period 2 in terms of consumption in period 1

Two equivalent ways of writing the first order condition:

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r} \quad (4) \quad \text{and} \quad u'(C_1) = \beta(1+r)u'(C_2) \qquad (4')$$

The consumption Euler equation: In optimum one cent yields the same (expected) return in terms of utility irrespective of whether it is spent on consumption now or invested and the proceeds spent on consumption next period.

Since $u'' < 0$, $C_2 > C_1$ if, and only if, $\beta(1+r)>1$

$\beta(1+r) > 1$ means that the interest rate exceeds the subjective discount rate.

$\beta(1+r) = 1 \;\Rightarrow\; C_1 = C_2 = C$. Complete consumption smoothing.

Convex preferences mean that there is always some tendency to consumption smoothing.

Example 1: $\beta(1+r) = 1,\ C_1 = C_2 = C$. Complete consumption smoothing.

Insertion in the budget constraint yields

$$C = \frac{(1+r)Y_1 + Y_2}{2+r} \qquad (5)$$

The current account in this case is

$$CA_1 = Y_1 - C = Y_1 - \frac{(1+r)Y_1 + Y_2}{2+r} = \frac{Y_1 - Y_2}{2+r} \qquad (6)$$

The main determinant of the current account is the difference between present and future income.

CA_1/Y_1 *depends only on* $(Y_1 - Y_2)/Y_1$ *not on the absolute level of income*

Example 2: CRRA utility function

$$u(C) = \frac{1}{1-\frac{1}{\sigma}} C^{1-\frac{1}{\sigma}} \qquad (7)$$

σ is the intertemporal elasticity of substitution.

$$u'(C) = C^{-1/\sigma}$$

Hence, the first order condition can be written

$$\frac{\beta u'(C_2)}{u'(C_1)} = \beta\left(\frac{C_2}{C_1}\right)^{-\frac{1}{\sigma}} = \frac{1}{1+r} \qquad (8)$$

or

$$\beta(1+r) = \left(\frac{C_2}{C_1}\right)^{1/\sigma} \Leftrightarrow \frac{C_2}{C_1} = \lfloor\beta(1+r)\rfloor^{\sigma} \Leftrightarrow C_2 = \lfloor\beta(1+r)\rfloor^{\sigma} C_1$$

C_2 is proportional to C_1 with the factor of proportionality increasing in r.

CRRA-example continued

From the first order condition and the budget equation

$$C_1 = \frac{Y_1 + (1+r)^{-1} Y_2}{1 + (1+r)^{-1}[\beta(1+r)]^{\sigma}} = \frac{(1+r)Y_1 + Y_2}{2 + r + \{[\beta(1+r)]^{\sigma} - 1\}} \qquad (9)$$

The current account is then

$$CA_1 = Y_1 - C_1 = \frac{Y_1 - Y_2 + \{[\beta(1+r)]^{\sigma} - 1\} Y_1}{2 + r + \{[\beta(1+r)]^{\sigma} - 1\}} \qquad (10)$$

Two motives for saving in the first period:

1. Consumption smoothing. Positive savings if $Y_1 > Y_2$
2. Rate of return. Positive savings if $\beta(1+r) > 1$, ($r > \delta$)

Strength of the last motive depends on the intertemporal substitution elasticity

Savings rate (CA / Y) is independent of income level

CRRA-example continued

$$C_1 = \frac{(1+r)Y_1 + Y_2}{1 + (1+r) + \{[\beta(1+r)]^\sigma - 1\}} \qquad (9')$$

Effects of *r* on C_1

1) The substitution effect related to $[\beta(1+r)]^\sigma$. Always negative.
2) Two opposing income effects (one in the numerator, one in the denominator).
    - Life-time income increases in proportion to Y_1
    - The real value of life time income decreases in proportion to C_1

    Net income effect is negative if $Y_1 < C_1$, positive if $Y_1 > C_1$

Hence, the effect of an increase in *r* on the current account is unambiguously positive if $Y_1 < C_1$, but may be negative if Y_1 is sufficiently greater than C_1 and the elasticity of substitution is low.

A high *r* makes it less expensive to smooth consumption when $Y_1 > Y_2$, *more expensive when* $Y_2 > Y_1$.

Summing up results on saving / current account

1. The absolute income level is not likely to be an important determinant of the CA.
2. Countries with high expected income growth should tend to have CA deficits, countries with low (negative) expected income growth to have surpluses.
3. Patient countries (with β close to 1) should tend to have current account surpluses, impatient ones to have deficits.

Does the model help explaining:

- The US deficit and the Chinese surplus?
- The surpluses of oil-rich countries?

More questions:

- Are we in the first period?
- How is the world interest rate determined?
- Government deficits? Investment?

Adding government consumption and taxes

Assumption: Government budget balanced in present value terms

$$T_1 + \frac{1}{1+r} T_2 = G_1 + \frac{1}{1+r} G_2 \qquad (11)$$

Budget constraint of the consumer

$$C_1 + \frac{C_2}{1+r} = (Y_1 - T_1) + \frac{Y_2 - T_2}{1+r} \;=\; Y_1 + \frac{Y_2}{1+r} - [T_1 + \frac{1}{1+r} T_2]$$

Or after inserting from (6)

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} - [G_1 + \frac{1}{1+r} G_2] \qquad (12)$$

Consumption Euler-equation unaffected provided that utility is separable in C and G:

$$u(C) + v(G)$$

In the example with $\beta(1+r) = 1$

$$C_1 = C_2 = C = \frac{(1+r)(Y_1 - G_1) + (Y_2 - G_2)}{2+r} \qquad (13)$$

The current account is then

$$CA_1 = Y_1 - C - G_1 = \frac{Y_1 - Y_2 - (G_1 - G_2)}{2+r} \qquad (14)$$

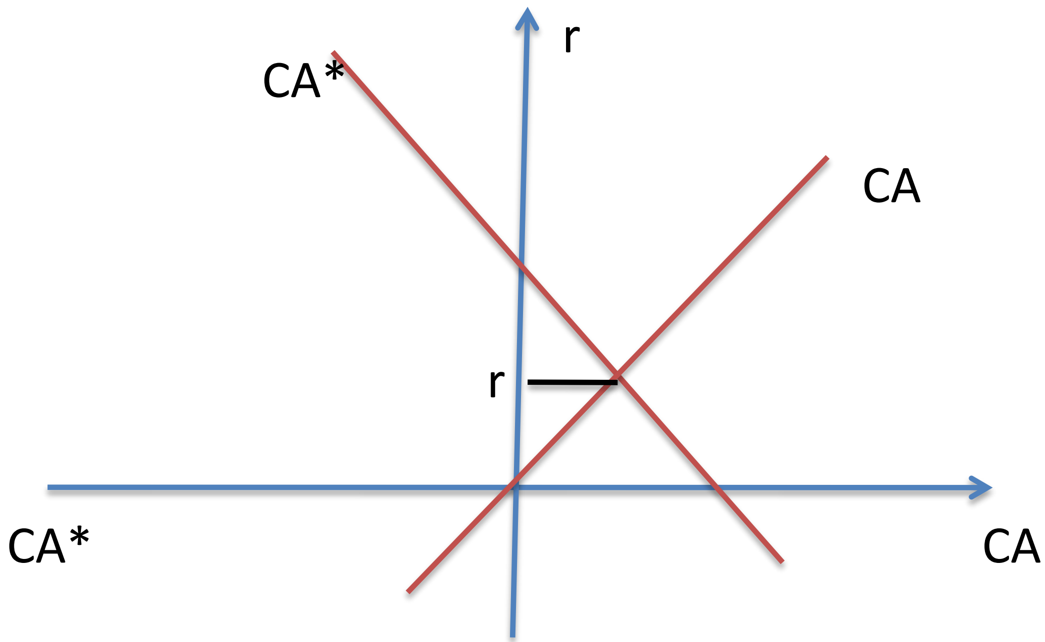
Temporarily high government expenditures now can produce a current account deficit.

Taxes and the size of deficits do not matter (because of Ricardian equivalence)

General equilibrium. Determination of r.

Two countries,(no explicit government sector)

Equilibrium condition $CA_1 = -CA_1^*$.



World equilibrium in period 1.

CRRA-example again

Assume same β in both countries

$$CA_1 = Y_1 - C_1 = \frac{Y_1 - \ Y_2 + \{[\beta(1+r)]^\sigma - 1\}Y_1}{2 + r + \{[\beta(1+r)]^\sigma - 1\}}$$

$$CA_1^* = Y_1^* - C_1^* = \frac{Y_1^* - Y_2^* \ + \{[\beta(1+r)]^\sigma - 1\}Y_1^*}{2 + r + \{[\beta(1+r)]^\sigma - 1\}}$$

Equilibrium condition

$$CA_1 = -CA_1^*$$

Equivalent to

$Y_1 - \ Y_2 + \{[\beta(1+r)]^\sigma - 1\}Y_1$ =-$Y_1^* + Y_2^* \ \{[\beta(1+r)]^\sigma - 1\}Y_1^*$

Which can be solved to yield

$$1 + r = \frac{1}{\beta}\left(\frac{Y_2 + Y_2^*}{Y_1 + Y_1^*}\right)^{1/\sigma}$$

The world real interest rate depends positively on the world growth rate and negatively on patience.

A low elasticity of substitution means that the growth rate has a strong effect on the interest rate.

Why then such low real interest rates after 2001?

Since by definition $\beta = 1/(1+\delta)$, where δ is the subjectiv disount rate, we can also write

$$1 + r = (1+\delta)\left(\frac{Y_2 + Y_2^*}{Y_1 + Y_1^*}\right)^{\frac{1}{\sigma}}$$

World consumption equals world output in each period. Hence,

$$\frac{C_2 + C_2^*}{C_1 + C_1^*} = \frac{Y_2 + Y_2^*}{Y_1 + Y_1^*} = \frac{Y_2^W}{Y_1^W}$$

Complete consumption smoothing is impossible.

$$1 + r = (1+\delta)\left(\frac{Y_2^W}{Y_1^W}\right)^{1/\sigma} \qquad (5)$$

- Positive output growth means $r > \delta$ and r > world growth rate if $\sigma < 1$
- Higher output growth means higher world interest rate
- Effect is stronger the lower is the elasticity of substitution

$$CA_1 > 0 \Leftrightarrow \frac{Y_2}{Y_1} < \frac{Y_2^W}{Y_1^W}$$

- Same consumption growth everywhere
- Countries with slow income growth need to save in first period
- Contries with high income growth want to borrow in first period

Including investment

Production function

$$Y_t = A_t F(K_t), \qquad t = 1,2 \qquad (6)$$

A_t an exogenous productivity factor

K_t capital stock at beginning of period t

Assumptions: $F' > 0,\ F'' < 0,\ F(0) = 0$

Assuming no depreciation, capital accumulates according to

$$K_t = K_{t-1} + I_{t-1}, \qquad t = 2,3 \qquad (7)$$

K_1 is given by past history, $K_3 = 0$ since the economy ends there

By implication: $I_2 = -K_2$

Period budget constraints

$$C_1 + B_2 + K_2 = Y_1 + K_1 \qquad (8)$$

$$C_2 = Y_2 + K_2 + (1 + r)B_2 \qquad (9)$$

Elimination of B_2 yields present value budget constraint

$$C_1 + \frac{C_2}{1+r} = K_1 + Y_1 + \frac{Y_2 - rK_2}{1+r} \qquad (10)$$

Separation of consumption and investment decisions:

1. Maximize total wealth (rhs of (10)). Since K_1 and $Y_1 = F(K_1)$ are given, this amounts to maximizing $Y_2 - rK_2$ with respect to K_2. Implicitly this also determines I_1.
2. Maximize utility with respect to C_1 and C_2 given total wealth. Same problem as before, same Euler equation.

Wealth maximization:

Max $Y_2 - rK_2 = A_2F(K_2) - rK_2$

First order condition:

$$A_2F'(K_2) = r \qquad (11)$$

Two ways of providing for the future:

- Lending to abroad, constant returns
- Investing in productive capital at home, diminishing return

Do the latter until returns are equalized.

Since $I_1 = K_2 - K_1$, I_1 depends negatively on *r* and K_1, positively on A_2

Effects of exogenous variables on the current account of a small open economy

$$CA_1 = S_1 - I_1 = A_1 F(K_1) - C_1 - I_1 = A_1 F(K_1) - C_1 - (K_2 - K_1) \qquad (12)$$

1) An increase in *r* now has three different types of effects on the CA:
   i) The usual positive substitution effect on savings.
   ii) Income effects on savings.

      Net borrowers ($B_1 < 0$) lose real income, consume less and save more.

      Net lenders ($B_1 > 0$) gain and save less.
   iii) A positive effect because an increase in *r* reduces investment demand I_1.

Total effect is ambiguous for net lenders; the investment effect iii) diminishes the ambiguity.

A digression on the income effects

Their sign can be found by looking at the consumer's budget constraint, conveniently rewritten as

$$(1+r)C_1 + C_2 = (1+r)(K_1 + Y_1) + Y_2 - rK_2 \ \text{ or}$$

$$(1+r)C_1 + C_2 = K_1 + (1+r)Y_1 + Y_2 - rI_1$$

Does an increase in *r* tighten or relax this constraint? (Does it increases the lhs more or less than the rhs given the initial values ofC_1, C_2,Y_1, Y_2 and I_1)?

Answer: An increase in r raises income more than expenditure if, and only if, $Y_1 - I_1 > C_1$ or $B_1 = Y_1 - C_1 - I_1 > 0$.

- An increase in r also changes K_2 and Y_2, but since we are starting from an optimum, this net effect of this on real income is zero (the envelope result).
- At the world level gains and losses from an increase in r are netted out.

Back to the other exogenous variables

2) An increase in A_1 works like an exogenous increase in Y_1 in the exchange economy. CA_1 is improved. No effect on investment.

3) An increase in A_2 now has two effects that lead to a deterioration of the current account:
   i) For a given K_2, Y_2 is increased. As in the endowment economy, this leads to increased $C_1$and a deterioration in CA_1.
   ii) From $A_2F'(K_2) = r$ we see that the optimal K_2 increases. Hence, I_1 increases and CA_1 deteriorates.

   The increase in K_2 that comes out of ii) has on the margin no net effect on income in period 2 since K_2 is optimized initially (the envelope theorem).

4) An increase in K_1 has two opposing effects

   i) It increases total wealth. Part of this is spent on C_1. Hence, CA_1 deteriorates, but less than the increase in K_1.

   ii) Since K_2 is unaffected, the increase in K_1 reduces I_1 and improves CA_1 one for one.

In this case the second effect obviously dominates. Countries with a high initial capital stock will *ceteris paribus* tend to have a CA surplus.

World equilibrium with investment

New opportunities:

- The sum of world output over the two periods can be increased by consuming less and investing more in the first period.
- The distribution of world consumption between the two periods can be smoothed by adjusting investment in the first period.

Equilibrium is characterized by

- *efficiency in production*

$$A_2F'(K_2) = A_2^*F'(K_2^*) = r \qquad (13)$$

Productive capital is distributed in a way that maximizes second period output

- *efficiency in distribution*

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{\beta^* u'(C_2^*)}{u'(C_1^*)} = \frac{1}{1+r} \qquad (14)$$

Consumers cannot increase utility by exchanging goods between them.

- *overall efficiency*

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{\beta^* u'(C_2^*)}{u'(C_1^*)} = \frac{1}{1+A_2F'(K_2)} = \frac{1}{1+A_2^*F'(K_2^*)} \qquad (15)$$

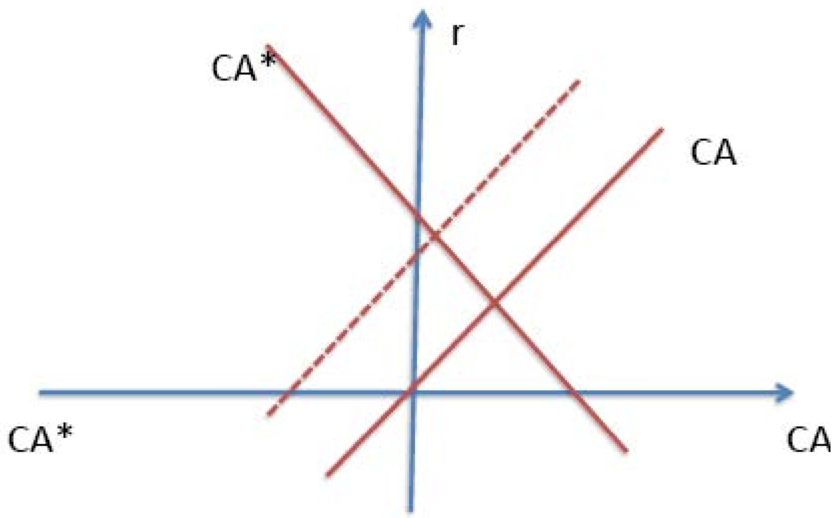
MRS = MRT No gain from moving output between the periods

Standard results on the efficiency of competitive equilibrium apply.

Standard results on the gains from trade apply.

New source of gains from trade: More efficient use of capital.

Two-country equilibrium, period 1



Increase in A_2 shifts CA_1 to the left.

$A_2 \uparrow \quad \rightarrow \quad r\uparrow, \quad CA\downarrow, \quad CA^*\uparrow$

$$A_2 F'(K_2) = r$$

Opposing effects on $I_1 = K_2 - K_1$ from A_2 and r.

- Increased return to investment
- More demand for C_1

Net effect on I_1 ambiguous, negative if strong desire for consumption smoothing.

K_2^* and $I_1^* \downarrow$, since r is up and A_2^* is unchanged.

How are Home and Foreign welfare affected by an increase in A_2?

All effects on Foreign come through the increase in *r.*

→ Foreign gains if it is a net lender, loses if it is a net borrower.

Home has in addition a direct positive effect from A_2.

→ If a net lender, home gains on both accounts

→ If a net borrower, home gains on A_2 and loses on *r*. Net effect ambiguous.

Immiserizing growth, most likely if

- Strong preference for smoothing of consumption (low σs)
- Low or no response of investment to interest rate
- High debt

Real problem or theoretical curiosity?

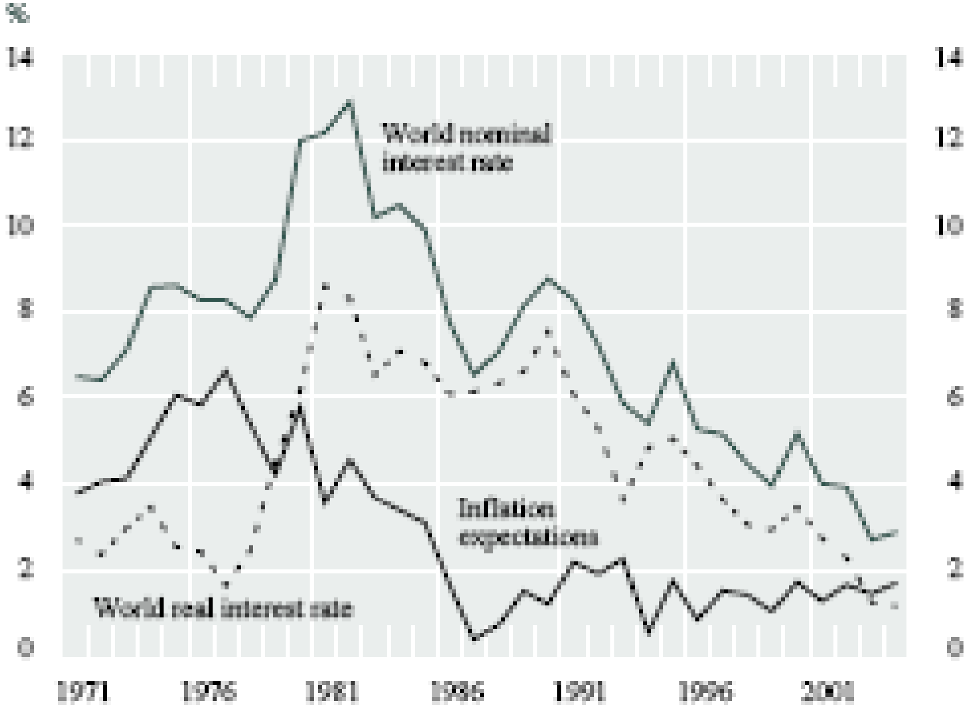
Current account, saving and investment in real capital in per cent of GDP 2006

Country	Current account	Saving	Investment
Germany	5,0	22,8	17,8
Japan	3,9	28,0	24,1
Developing Asia	6,1	43,9	37,9
United Kingdom	-3,2	14,8	18,0
United States	-6,2	14,1	20,0
World	0,3	23,3	23,0

The figures are for *gross* saving and *gross* investment

Chart 1

World Interest Rates and Inflation Expectations



Source: World Bank, BIS, IMF, Bank of Canada calculations

Bank of Canada Review • Winter 2006–2007