

The current account in a Ramsey type models

Econ 4330 International Macroeconomics Spring2014

Second lecture, part 1

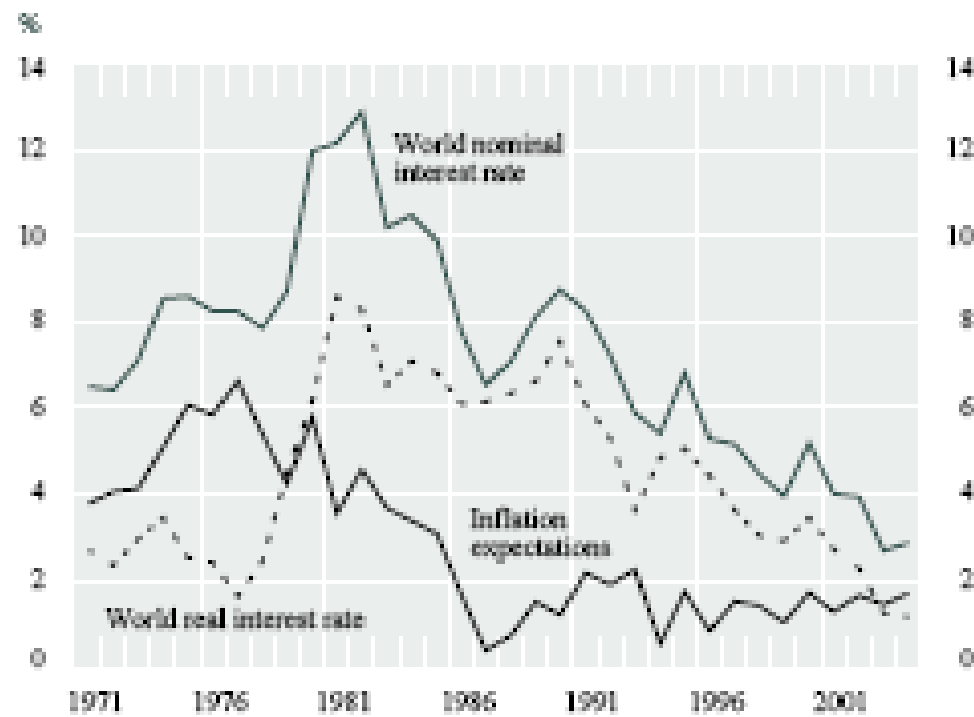
Asbjørn Rødseth January 21 2011

Current account, saving and investment in real capital in per cent of GDP 2006

Country	Current account	Saving	Investment
Germany	5,0	22,8	17,8
Japan	3,9	28,0	24,1
Developing Asia	6,1	43,9	37,9
United Kingdom	-3,2	14,8	18,0
United States	-6,2	14,1	20,0
World	0,3	23,3	23,0

The figures are for *gross* saving and *gross* investment

Chart 1
World Interest Rates and Inflation Expectations



Source: World Bank, BIS, IMF, Bank of Canada calculations

The current account in a Ramsey-type model of a small open economy

- Real interest rate, r , given from abroad
- The growth rate of the economy is below r , or expected to be below r in the long run
- Single commodity
- Representative consumer with infinite horizon
- No uncertainty, perfect foresight

Obstfeld and Rogoff Ch2

Budget constraint

Finite horizon:

Loans have to be paid back with interest in the end

Infinite horizon:

Loans can be rolled over ad infinitum as long as interest is paid

Foreign assets evolves according to

$$B_{s+1} - B_s = rB_s + Y_s - (C_s + G_s + I_s), \quad s = t, t+1, t+2, \dots \quad (1)$$

- B_s is net foreign assets at end of period $s - 1$

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The present value constraint:

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- $\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s + G_s) \leq (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} Y_s \quad (2)$
- PV of expenditure should not exceed initial asset plus PV of income.
- Assuming:
PV of income exists – always true if after some point in the future interest rates always exceed growth rates, or
$$\frac{Y_{s+1}}{Y_s} < (1+r) \text{ for all } s \text{ greater than some } T$$

Upper limit on foreign debt:

$$B_t \geq -\frac{1}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (\hat{Y}_s - \hat{I}_s)$$

- Zero consumption $C_s = G_s = 0$
- Present value of net output maximized gives \hat{Y}_s, \hat{I}_s
- All income to service debt
- If net output grows with rate γ , the debt limit grows in the same proportion.
- Current account deficit accepted: γB_t
- Trade surplus required $(r - \gamma)(-B_t)$

How large trade surpluses are achievable?

- Default risk may give rise to a lower debt limit
- Debt limits are on individual borrowers, not on nations
- Household constraint

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_{-s} \leq (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (\hat{Y}_s - \hat{I}_s G_s) = (1+r)W_t$$

Optimal behavior

$$U_t = u \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \quad (4)$$

Production functions:

$$Y_t = A_t F(K_t), \quad t = 1, 2, \dots \quad (5)$$

Accounting relations:

$$K_t = K_{t-1} + I_{t-1}, \quad t = 1, 2, \dots \quad (6)$$

$$B_{t+1} - B_t = rB_t + Y_t - C_t - I_t - G_t, \quad t = 1, 2, \dots \quad (7)$$

+ Present value constraint

First-order conditions

Consumption Euler equation

$$u'(c_t) = \beta(1 + r)u'(c_{t+1})$$

Marginal productivity condition

$$F'(K_t) = r$$

Consumption and investment decisions can be separated

Paths for investment and the capital stock can be found by maximizing the present value of firms.

CRRA example: Solving for C_t

- $u(C) = \frac{1}{1-\frac{1}{\sigma}} C^{1-\frac{1}{\sigma}} \quad (8)$
- Euler equation reduced to
-
- $C_{s+1} = [\beta(1+r)]^\sigma C_s = (1+v)C_s$
-
- $v = [\beta(1+r)]^\sigma - 1$ is the growth rate of consumption.
- Hence, $C_s = (1+v)^{s-t} C_t$,
- Consumption grows if $\beta(1+r) > 1$
- Is $v < r$? Yes, always when $\beta < 1$ and $\sigma \leq 1$ (and maybe even when $\sigma > 1$).

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CRRA Solution

From present value constraint

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} C_s = (1+r)W_t$$

After inserting from Euler-equation:

$$\sum_{s=t}^{\infty} \left(\frac{1+\nu}{1+r} \right)^{s-t} C_t = \frac{1+r}{r-\nu} C_t = (1+r)W_t$$

$$C_t = (r-\nu)W_t \quad (9)$$

$\nu = 0$ Consumption equals permanent income

$\nu > 0$ Consume less than permanent income

Decomposing the trade balance

Permanent values

$$\tilde{Y}_t = \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} Y_s$$

Total wealth rewritten

$$W_t = B_t - \frac{1}{r} (\tilde{Y}_t - \tilde{I}_t - \tilde{G}_t) \quad (10)$$

Consumption:

$$C_t = rB_t + \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t - vW_t \quad (11)$$

Current account:

$$CA_t = Y_t - \tilde{Y}_t - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) + \frac{v}{1+r} W_t \quad (12)$$

$$CA_t = rB_t + Y_t - C_t - I_t - G_t \quad (13)$$

Does it work?

- Fast growth vs low growth, China vs US
- Capital rich vs capital poor
- Demand shocks vs supply shocks
- Different preferences yields extreme results.

Overlapping generations and life-cycle saving

International Macro: Lecture 4

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Motivation

- ▶ Individuals have finite lives
- ▶ The economy persists
- ▶ Individual decision making, not dynastic

Different from infinite horizon model

- ▶ Positive relation between savings and growth
- ▶ Wealth to GDP ratios are bounded
- ▶ Timing of taxes matter for current accounts

Main assumptions

- ▶ Small open economy
- ▶ Output exogenous (endowment economy)
- ▶ Given world interest rate
- ▶ Consumers live for two periods
- ▶ Generations overlap
- ▶ No bequests and no gifts from children
- ▶ One representative consumer for each generation

Behavior of individual consumer

Utility function, generation borne at t :

$$U = u(c_t^Y) + \beta u(c_{t+1}^O) \quad (1)$$

Budget constraint:

$$c_t^Y + (1+r)^{-1}c_{t+1}^O = y_t^Y + (1+r)^{-1}y_{t+1}^O = w_t \quad (2)$$

w_t total life-time wealth of individual of generation t

First order condition:

$$u'(c_t^Y) = \beta(1+r)u'(c_{t+1}^O) \quad (3)$$

Example: log utility

$$u(c) = \ln c \quad (4)$$

Special case of CRRA-utility with $\sigma = 1$

First order condition:

$$1/c_t^Y = \beta(1+r)/c_{t+1}^O$$

or

$$c_{t+1}^O = \beta(1+r)c_t^Y$$

Insert in budget equation, solve and get:

$$c_t^Y = \frac{w_t}{1+\beta}, \quad c_{t+1}^O = \frac{\beta(1+r)w_t}{1+\beta} \quad (5)$$

Saving when young

Individual saving (use (5) and (2)):

$$s_t^Y = y_t^Y - c_t^Y = \frac{1}{(1+\beta)(1+r)} \left[\beta(1+r)y_t^Y - y_{t+1}^O \right] \quad (6)$$

e = the growth rate of income from young to old,

$$y_{t+1}^O = (1+e)y_t^Y.$$

$$s_t^Y = \frac{1}{(1+\beta)(1+r)} [\beta(1+r) - (1+e)] y_t^Y$$

Savings rate of the young is then:

$$\mu = s_t^Y / y_t^Y = \frac{[\beta(1+r) - 1] - e}{(1+\beta)(1+r)} \quad (7)$$

Saving when young

$$\mu = \frac{[\beta(1+r) - 1] - e}{(1+\beta)(1+r)}$$

Two reasons for saving:

- ▶ the return is high enough to overcome impatience $\beta(1+r) > 1$.
- ▶ income is lower when old $e < 1$

Retirement creates need for saving.

Saving when old

Saving when old is the negative of saving when young:

$$s_{t+1}^O = -s_t^Y$$

The sum of saving over the individual life-cycle is zero
+ Standard assumption $y^O \ll y^Y$

\Rightarrow

The young are saving, the old are dissaving.

Aggregate saving

The young save, the old dissave

+ Sum of savings over individual life-cycle is zero

\Rightarrow

Aggregate savings positive only if young are richer or more numerous than old.

Aggregation

Total savings

$$S_t = N_t s_t^Y + N_{t-1} s_t^O \quad (8)$$

N_t Size of young generation at t

Total financial assets of households at end of period t :

$$B_{t+1}^P = N_t s_t^Y \quad (9)$$

Total household income:

$$Y_t = N_t y_t^Y + N_{t-1} y_t^O \quad (10)$$

Growth and savings

n growth rate of population $N_{t+1} = (1 + n)N_t$

g growth rate of income between generations

$$y_{t+1}^Y = (1 + g)y_t^Y$$

e growth rate of income over life-cycle,

$$y_{t+1}^O = (1 + e)y_t^Y$$

$$\begin{aligned} S_t &= N_t \mu y_t^Y - N_{t-1} \mu y_{(t-1)}^Y \\ &= N_t \mu y_t^Y - N_t (1 + n)^{-1} \mu y_t^Y (1 + g)^{-1} \\ &= \mu N_t y_t [(1 + n)(1 + g) - 1] / [(1 + n)(1 + g)] \quad (11) \end{aligned}$$

No growth, no net saving

Growth and savings cont.

Aggregate output:

$$Y_t = N_t y_t^Y + N_{t-1} y_t^O = N_t y_t \frac{(1+n)(1+g) + (1+e)}{(1+n)(1+g)} \quad (12)$$

Aggregate savings rate:

$$\begin{aligned} \frac{S_t}{Y_t} &= \mu \frac{(1+n)(1+g) - 1}{(1+n)(1+g) + (1+e)} \\ &= \left(\frac{[\beta(1+r) - 1] - e}{(1+\beta)(1+r)} \right) \left(\frac{n+g+ng}{2+n+g+ng+e} \right) \quad (13) \end{aligned}$$

(Compare p. 150 in OR, where $n = 0$ and $\beta(1+r) = 1$).

Growth and savings cont.

Focus on case where $e < 0$ and $\beta(1+r) = 1$

$$\mu = \frac{-e}{(1+\beta)(1+r)} = \frac{-\beta e}{1+\beta}$$

$$\frac{S_t}{Y_t} = -e \left(\frac{n+g+ng}{2+n+g+ng+e} \right) \frac{\beta}{1+\beta}$$

Savings rate is

- ▶ decreasing in e
- ▶ Increasing in n
- ▶ Increasing in g

Life-cycle model and timing of taxes

- ▶ Infinite horizon consumers: Compensates for tax reduction by saving more because they know they have to pay-back through higher taxes later
- ▶ Life-cycle consumer: Spends the part of the tax reduction that is going to be paid by future generations

Life-cycle model - evaluation

- ▶ Can explain that fast growing countries save more
- ▶ Net foreign assets stays within limits
- ▶ Retirement saving
- ▶ Precautionary saving
- ▶ Borrowing constraints
- ▶ Other life-cycle related motives
- ▶ Bequests without dynastic optimization

Investment and growth

Production function (constant returns):

$$Y = F(K, AN) = ANF(K/AN, 1) = ANf(k) \quad (14)$$

$$k = K/AN$$

First order condition:

$$f'(k) = r \quad (15)$$

k constant when r constant

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} = \frac{K_t}{A_tN_t} = k \quad (16)$$

Investment and growth

$$K_{t+1} = K_t \frac{A_{t+1} N_{t+1}}{A_t N_t} = K_t (1 + g)(1 + n) \quad (17)$$

$$I_t = K_{t+1} - K_t = [(1 + g)(1 + n) - 1]K_t = (g + n + gn)K_t \quad (18)$$

- ▶ Investment rate high in fast-growing economies
- ▶ Feldstein-Horioka puzzle

Determination of world real interest rate

- ▶ Two countries, same population, same productivity level
- ▶ Same growth rates g , n and e and same β
- ▶ No labor income when old $e = -1$
- ▶ The savings rate of the young is $\mu = \beta/(1 + \beta)$ in both countries.
- ▶ Production functions are Cobb-Douglas with the same capital share α :

$$Y_t = (A_t N_t)^{1-\alpha} K_t^\alpha$$

- ▶ Labor's share of output is $1 - \alpha$
- ▶ Focus on steady state, $k_t = k$, constant

Determination of world real interest rate 2

Equilibrium in world capital markets require

$$K_{t+1}^W = S_t^{WY} \quad (19)$$

Capital stock carried over from t to $t + 1$ equals savings of the young at the end of period t .

Savings of the young equal their savings rate times labor income:

$$S_t^{WY} = \mu Y_t^{YW} = \mu(1 - \alpha) Y^W = \mu(1 - \alpha) A_t N_t^W f(k_t) \quad (20)$$

Last two equations combined:

$$K_{t+1}^W = \mu(1 - \alpha) A_t N_t^W f(k_t) \quad (21)$$

Divide by $A_{t+1} N_{t+1}^W$ on both sides:

Determination of world real interest rate 3

$$\frac{K_{t+1}^W}{A_{t+1}N_{t+1}^W} = \mu(1 - \alpha) \frac{A_t N_t^W}{A_{t+1}N_{t+1}^W} f(k_t)$$

or

$$k_{t+1} = \frac{\beta}{1 + \beta} \frac{1 - \alpha}{(1 + n)(1 + g)} k_t^\alpha \quad (22)$$

Steady state capital intensity, \bar{k} is given by

$$\bar{k} = \frac{\beta}{1 + \beta} \frac{1 - \alpha}{(1 + n)(1 + g)} \bar{k}^\alpha \quad (23)$$

Solve equation (23) for \bar{k} :

$$\bar{k} = \left(\frac{\beta(1 - \alpha)}{(1 + \beta)(1 + n)(1 + g)} \right)^{1/(1 - \alpha)} \quad (24)$$

The world real interest rate in steady state

$$\bar{r} = f'(\bar{k}) = \alpha \bar{k}^{\alpha-1} = \frac{\alpha(1+\beta)(1+n)(1+g)}{\beta(1-\alpha)} \quad (25)$$

\bar{r}

- ▶ is increasing in the growth rates
- ▶ is decreasing in β (patience)
- ▶ is increasing in α (capital's share)
- ▶ may be on either side of natural growth rate $n + g + ng$

Real interest rate below natural rate of growth

- ▶ Government of small open economy can
 - ▶ borrow now
 - ▶ spend proceeds on current old
 - ▶ roll over the loan forever
 - ▶ pay interest with new loans
- ▶ Many governments tempted by this
- ▶ Savings of the young invested in gov. bonds
- ▶ Real capital formation crowded out
- ▶ World interest rate will increase
- ▶ Government borrowing may bring world interest rate above growth rate

Saving in corporations

- ▶ Norway 2008: Saving in the corporate sector six times saving in the household sector
- ▶ China 2008: 44 per cent of savings from household sector, 35 from corporate, 20 from government
- ▶ Households own corporations
- ▶ Governments own corporations
- ▶ Importance of income distribution for savings