Problem set 1

ECON 4330

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We are looking at an open economy that exists for two periods. Output in each period Y_1 and Y_2 respectively, is given exogenously. A representative consumer maximizes life-time utility

$$U = u(C_1) + \beta u(C_2)$$

where C_1 and C_2 are consumption in the two periods and β is a subjective discount factor, $0 < \beta < 1$.The country can borrow and lend in world markets at a given real interest rate, r. The initial asset is zero. Hence, the budget constraint can be written

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

1. Derive the first order condition for optimal consumption and interpret it.

2. Derive the welfare effects of an increase in r, i.e dU/dr. Provide intuition. (use the envelope condition)

3. Assume CRRA utility
$$u(C) = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$
Find an expression for date 1 consumption and current account as functions of exogenous variables (Y_1, Y_2, r).

4. Assume $Y_2 = 0$

   (a) Derive $\partial C_1/\partial r$ and find condition that makes sure the current account is improving when the world interest rate goes up.

   (b) Describe how C_1 responds to changes in r in terms of substitution and income effects.

   (c) What additional effect comes in if we assume $Y_2 > 0$?

5. Suppose a foreign country has the same preferences as the home country, equal date 1 output $Y_1^* = Y_1$ but different date 2 output Y_2^*

(a) Assume higher income growth in the home country, i.e. $Y_2 > Y_2^*$. Derive the autarky interest rate in both countries and compare.

(b) Suppose the world market consists of these two countries. State the equilibrium condition, and show in a graph how the interest rate will be determined. Which country will run a current account surplus in period 1? Intuitively, what are the gains from trade?

(c) Using the answer from question 2. what happens to welfare in the home country if the foreign country's output growth increases (Y_2^* up)?

6. We now extend the model to include a production side. Each country has access to the same technology and the technology does not change. The production function is $Y = F(N, K)$ where N and K are respectively the inputs of labor and capital. The production function is homogeneous of degree one and has standard neoclassical properties. The labor input is given exogenously and is the same in both countries and both periods. Each country has inherited a capital stock from the past, K_1 and K_1^*respectively, that can be used in production in period 1. The capital stock can be augmented by investment in period 1, which then adds to the input of capital in period 2, K_2 and K_2^*. At the end of period 2 the remaining capital stock is consumed. The budget constraint of the home country can then be written

$$C_1 + I_1 + \frac{C_2 + I_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}$$

where $I_1 = K_2 - K_1$ and $I_2 = -K_2$. Explain how the home country's investment demand in period 1 is determined and what the inclusion of capital means for the relationship between the world interest rate and the home country's current account in the first period.

7. Compare the capital stocks of the two countries in the second period. Suppose the home country has inherited more capital than the foreign country ($K_1 > K_2$). What does this imply for the current accounts in the two periods?

8. Finally, suppose international borrowing and lending had not been possible. From the point of view of wage earners in the home country, would this be an advantage or a disadvantage?