An infinite horizon model

In this problem we consider an infinite horizon model with a representative agent and perfect foresight. Each period, the agent must obey the following budget constraint:

$$C_s + B_{s+1} = Y_s + (1+r)B_s$$

1. Based on the fact that the budget constraint holds for every period from t to $t+T$, show that this implies

$$(1+r)B_t = \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (C_s - Y_s) + \frac{B_{t+T+1}}{(1+r)^T}$$

2. Explain the intuition behind

$$\lim_{T\to\infty} \frac{B_{t+T+1}}{(1+r)^T} = 0$$

3. Impose this restriction and assume that $C_s = cY_s$ and $Y_s = (1+g)^s Y_t$.

   (a) Find the intertemporal budget constraint for this case (when $g < r$).

   (b) Imagine that keeping consumption at a fixed share c of output indeed is the optimal consumption-choice of a representative agent. Is c above or below one?

   (c) Assume $g = 0$. What does the time-profile of B_t look like for a given value of c?

   (d) Assume $g > 0$ (but also $g < r$). What does the time-profile look like now?

4. Now assume (as in question 6 of the first problem) that output is a function of the capital stock, $Y_t = A_t F(K_t)$. The utility function is specified as $U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$. Use the period s budget constraint to insert for C_s in the utility function.

   (a) Find the first-order condition with respect to K_{t+1} and B_{t+1}

   (b) Suppose productivity is constant $A_s = A_t$ for all $s \geq t$ and that, by coincidence, $\beta(1+r) = 1$. Describe the time-profiles of consumption, investment and the current account (you can assume that initial net foreign assets, B_t, are zero).

(c) Sketch the effects on consumption, investment and the current account from

i. An unexpected temporary increase in productivity in period $t+1$ (that only lasts one period)

ii. A temporary increase in productivity in $t+1$ (that only lasts one period) that becomes known at the beginning of period t

iii. An unexpected permanent increase in productivity