Seminar 4 note
ECON 4330

April 7, 2015

The value V depends on K_2, i.e., it depends on how much the country decides to invest. Assume that the country chooses K_2 to maximize $F(K_2) - K_2 - V(D, K_2)$. Find the first-order condition for optimal K_2. Give it an interpretation. HINT: Use the Leibniz rule.

Solution

Assuming that utility is linear in consumption, and $\beta(1+r) = 1$, expected utility maximization implies maximizing present value of income (we also set $r = 0$)The first order condition is:

$$F'(K_2) - 1 - \frac{\partial V}{\partial K_2} = 0$$

The first to terms represents the standard intertemporal trade off between current and future consumption, whereas the last term $\frac{\partial V}{\partial K_2}$ captures the effect on expected repayment from higher investment:

$$\begin{aligned}
\frac{\partial V}{\partial K_2} &= \eta F'(K_2) \int_{A_L}^{A^*} A\pi(A)dA + \eta F(K_2)A^*\pi(A^*)\frac{\partial A^*}{\partial K_2} - D\pi(A^*)\frac{\partial A^*}{\partial K_2} \\
&= \eta F'(K_2) \int_{A_L}^{A^*} A\pi(A)dA + (\eta F(K_2)A^* - D)\,\pi(A^*)\frac{\partial A^*}{\partial K_2} \\
&= \eta F'(K_2) \int_{A_L}^{A^*} A\pi(A)dA + \left(\eta F(K_2)\frac{D}{\eta F(K_2)} - D\right)\pi(A^*)\frac{\partial A^*}{\partial K_2} \\
&= \eta F'(K_2) \int_{A_L}^{A^*} A\pi(A)dA > 0
\end{aligned}$$

The default option acts as an implicit tax on investment, since creditors confiscate a fraction η of the return in the event of default, thereby depressing investment.