Problem set 3
ECON 4330

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Real exchange rates

To introduce the real exchange rate we need more than our one homogeneous good. We now introduce two sectors producing traded and non-traded goods.

1. Output in the two sectors are (subscripts T and N is for traded and non-traded):

$$Y_T = A_T K_T^{\gamma} L_T^{1-\gamma} \tag{1}$$

$$Y_N = A_N K_N^{\alpha} L_N^{1-\alpha} \tag{2}$$

Firms problems:

$$\max_{K_T, L_T} A_T K_T^{\gamma} L_T^{1-\gamma} - wL_T - rK_T \tag{3}$$

$$\max_{K_N, N_T} pA_N K_N^{\alpha} L_N^{1-\alpha} - wL_N - rK_N \tag{4}$$

(a) Find the first order conditions of the firms optimization problem in the two sectors when capital can be rented at an international market at price r and labor is mobile between sectors at home. The price of the non-traded good is p. The traded good is numeraire. Use capital intensities, $k_T = \frac{K_T}{L_T}$ and $k_N = \frac{K_N}{L_N}$, in the conditions.

Solution

$$\begin{aligned} \gamma A_T k_T^{\gamma-1} &= r \\ (1-\gamma) A_T k_T^{\gamma} &= w \\ p\alpha A_N k_N^{\alpha-1} &= r \\ p(1-\alpha) A_N k_N^{\alpha} &= w \end{aligned} \tag{5}$$

(b) Solve for the wage rate in the traded sector. How does it depend on the world interest rate?

Solution

Solve for k_T from the first condition and insert in the second.

$$\begin{aligned} k_T &= \left(\frac{r}{\gamma A_T}\right)^{\frac{1}{\gamma-1}} \\ w = (1-\gamma)A_T^{\frac{1}{1-\gamma}} &\left(\frac{r}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \end{aligned} \tag{6}$$

When the interest rate increase less capital is employed and we reduce the productivity of labor so wages decrease.

(c) Solve for the price of the non-traded good, p.

Solution

Start by finding p from each of the two last conditions and equate them to find k_N. Then insert that capital intensity into one of the expressions for p.

$$\begin{aligned} k_N^{1-\alpha}\frac{r}{A_N\alpha} = p = k_N^{-\alpha}&\frac{w}{A_N(1-\alpha)} \\ k_N &= \frac{w}{r}\frac{\alpha}{1-\alpha} \\ &.. \\ p = \frac{r}{A_N\alpha}&\left(\frac{w}{r}\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \\ &.. \\ p = \frac{r^\alpha}{A_N\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}&\left[(1-\gamma)A_T^{\frac{1}{1-\gamma}}\left(\frac{r}{\gamma}\right)^{\frac{\gamma}{1-\gamma}}\right]^{1-\alpha} \end{aligned} \tag{7}$$

(d) Let the real EXCHANGE rate be defined as $Q = \frac{EP^*}{P}$ and the price index as $P = (1)^\lambda p^{1-\lambda}$, where λ is the weight on traded good in the index. Assume $E = 1$(Euro?) and $P^* = 1$ and find out how Q depends on A_T, A_N and r.

Solution

A_T: Increased productivity in the traded sector will increase wages, wages must be equal in both sectors. When wages increase prices in the non-traded sector must increase to keep profitability. Higher prices in the home country result in a lower real exchange rate, Q, you give up less home goods to trade for foreign goods.

A_N: Increased productivity in the non-traded sector lower prices. Lower prices at home increase the real exchange rate.
r: Let's start with the expression we have above and find an easier expression for p as a function of r:

$$\begin{aligned}
p =& \frac{r^{\alpha}}{A_N \alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left[(1-\gamma)A_T^{\frac{1}{1-\gamma}}\left(\frac{r}{\gamma}\right)^{\frac{\gamma}{1-\gamma}}\right]^{1-\alpha} \\
p =& \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}A_N}\left[(1-\gamma)A_T^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}}\right]^{1-\alpha} r^{\alpha+\frac{\gamma(1-\alpha)}{\gamma-1}} \\
\text{Introduce: } B =& \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}A_N}\left[(1-\gamma)A_T^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}}\right]^{1-\alpha} \\
\text{and we have: } p =& Br^{\frac{\alpha-\gamma}{1-\gamma}}
\end{aligned} \tag{8}$$

The derivative with respect to r will then be:

$$\frac{\partial p}{\partial r} = B\frac{\alpha-\gamma}{1-\gamma}r^{\frac{\alpha-1}{1-\gamma}} \tag{9}$$

which is positive as long as α is larger than γ and gamma is less than one (B is positive under normal assumptions). The intuition is as follows; when the interest rate increase, that has two effects on domestic prices. One: It will decrease capital intensity in the traded sector and thus lower wages in both the traded and in the non-traded sector. This decrease the costs of production and thus the price. The second effect is that capital as a factor is more expensive and the will increase the price directly. Which effect is stronger is determined by the relative productivity of capital in the two sectors. If $\alpha > \gamma$, capital is more productive in the non-traded sector and the direct effect will dominate and prices will rise. If $\gamma > \alpha$, capital is more productive in the traded sector and the indirect effect through wages will dominate.