Problem set 3
ECON 4330

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Short look at OLG

This might not be covered in class, but the model should be known so we do a simple, but instructive, exercise.

1. We use a simple OLG model where consumers live for two periods. They have exogenous labor income only in the first period and save for old age at a world interest rate r. The instantaneous utility function is log utility and the problem of generation t is given by:

$$\begin{aligned} \max_{c_t^y, c_{t+1}^o} \quad & U_t = log(c_t^y) + \beta log(c_{t+1}^o) \\ s.t \quad & c_t^y + b_{t+1} = y_t \\ & c_{t+1}^o = (1+r)b_{t+1} \end{aligned} \tag{1}$$

   (a) Solve for the optimal savings for generation t.

   Assume that income grows by a fixed rate, g, every period such that $y_t = (1+g)^t y_0$. Solve for the net savings(savings of the young minus dis-savings of the old) in each period.

   (b) How does the time-profile of the net savings of the country depend on the growth rate?

   (c) How does this compare to the infinite horizon case from the last seminar?

Real exchange rates

To introduce the real exchange rate we need more than our one homogeneous good. We now introduce two sectors producing traded and non-traded goods.

2. Output in the two sectors are (subscripts T and N is for traded and non-traded):

$$Y_T = A_T K_T^{\gamma} L_T^{1-\gamma} \tag{2}$$

$$Y_N = A_N K_N^{\alpha} L_N^{1-\alpha} \tag{3}$$

Firms problems:

$$\max_{K_T, L_T} A_T K_T^{\gamma} L_T^{1-\gamma} - wL_T - rK_T \tag{4}$$

$$\max_{K_N, N_T} pA_N K_N^{\alpha} L_N^{1-\alpha} - wL_N - rK_N \tag{5}$$

   (a) Find the first order conditions of the firms optimization problem in the two sectors when capital can be rented at an international market at price r and labor is mobile between sectors at home. The price of the non-traded good is p. The traded good is numeraire. Use capital intensities, $k_T = \frac{K_T}{L_T}$ and $k_N = \frac{K_N}{L_N}$, in the conditions.

   (b) Solve for the wage rate in the traded sector. How does it depend on the world interest rate?

   (c) Solve for the price of the non-traded good, p.

   (d) Let the real interest rate be defined as $Q = \frac{EP^*}{P}$ and the price index as $P = (1)^{\lambda} p^{1-\lambda}$, where λ is the weight on traded good in the index. Assume $E = 1$(Euro?) and $P^* = 1$ and find out how Q depends on A_T, A_N and r.

3. Here we discuss the connection between the law of one price and absolute purchasing power parity(PPP).

   (a) First, explain why the law of one price implies PPP.

   (b) Now, show that PPP does not imply the law of one price. Hint: Start with the definitions of the law of one price and PPP and construct an example with three different goods.