Seminar exercises fall 2013

Instructions:

For all students: Please try to solve these exercises before the seminars. Solving problems is by far the best way to learn the material and prepare for the exam. It is generally a better idea to work a little with all questions than to work a lot with only a few of them.

For the first seminar meeting 27 August, no solution will be circulated on beforehand. This will change for the remaining five meetings. Exercises for these will be distributed before 27 August.

Exercises for seminar no. 1, 27 August

(1)

Consider the function $U(C) \equiv ae^{bC} + d$, where a, b, and d are constants, and $e \approx 2.718$ is the well-known constant. C is consumption.

1(a)

What condition(s) must be satisfied by a, b, and d in order for $E[U(C)]$ to properly represent the preferences of a risk averse person who maximizes von Neumann-Morgenstern expected utility? What are the coefficients of absolute and relative risk aversion, $R_A(C)$ and $R_R(C)$, for this U function?

Consider an individual planning for a future period, with the given U function with that/those conditions you stated in part 1(a). The individual has wealth Y_0 to be divided between two financial investments, Y_f (risk free) and Y_r (risky). The future consumption is equal to the sum of the future values of these investments. Y_f will increase by the factor $R_f \equiv 1 + r_f$, where r_f is the risk free interest rate. Y_r will increase similarly by the factor $\tilde{R} \equiv 1 + \tilde{r}$, where $\tilde{r}$ is a risky rate of return (— this may in fact be a decrease for low outcomes of $\tilde{r}$). The individual regards Y_0, R_f, and the probability distribution of $\tilde{R}$ as exogenously given.

In what follows, you should discuss both the case in which short selling is allowed and the case in which it is not.

1(b)

Describe the individual's maximization problem and its solution.

1(c)

Discuss the statement "Optimal Y_r does not depend on the size of Y_0."

Assume in the following that $\tilde{R}$ has only two possible outcomes, R_1 and R_2, and that $R_1 > R_2$. The probability of R_1 is p.

1(d)

Describe the solution to the maximization problem in this case. Show that under some conditions the optimal Y_r can be written as

$$Y_r^* = \frac{\ln \frac{p(R_1 - R_f)}{(1-p)(R_f - R_2)}}{-b(R_1 - R_2)}.$$

1(e)

Find out whether Y_r^* as given by the formula above is increased or decreased by changes in p, R_f, and b. Try to give intuitive explanations for these effects.

1(f)

Could Y_r^* from the formula exceed Y_0? What would the individual do then if borrowing is not allowed?

1(g)

Could Y_r^* from the formula be negative? What would the individual do then if short selling is not allowed?

1(h)

Maintain the assumption that $R_2 < R_1$. What will happen if also $R_1 < R_f$? What will happen if instead $R_2 > R_f$? Can these situations occur?

1(i)

What would be the solution for parts 1(a) – 1(d) if the individual is instead attracted to risk?

(2)

(Exam ECON4515 Spring 2006, problem 2, slightly rewritten and expanded)

There are two periods: period zero (today) and period 1 (next period). In the next period there are three possible states of the world: recession (with probability 0.3), normal growth (with probability 0.4), and expansion (with probability 0.3). There are two stocks (stock A and stock B) in the economy. The stocks will yield payoffs next period according to the following table:

Stock	Recession	Normal Growth	Expansion
A	6	12	14
B	15	5	10

Questions (a) and (b) use the concepts of stochastic dominance of the first and second order (D&D, sect. 4.6). In case this was not covered in the first lecture, you may still be able to solve questions (c) and (d) further down, which do not use those concepts.

(a) Can you rank the stocks by the criterion of first order stochastic dominance? If you can rank them, which stock first order stochastically dominates the other? You may use a graph to answer this question if you choose to.

(b) Can you rank the stocks by the criterion of second order stochastic dominance? If you can rank them, which stock second order stochastically dominates the other? You may use a graph to answer this question if you choose to.

Consider an investor with the following utility function:

$$u(w) = \sqrt{w}.$$

After spending what is needed for consumption today, the investor has a current wealth of 10 plus one share of stock A. The investor can either keep the share until next period or sell it now. Apart from the share, savings will earn a risk free interest rate of zero (for simplicity).

(c) What is the minimum price the investor would be willing to sell the stock for? Is this more or less than the expected payoff from this share of stock? Why?

Assume that the current wealth is 10000 plus one share of stock A, instead of 10 plus one share of stock A.

(d) What is the minimum price the investor would be willing to sell the stock for? How do you explain the difference between this answer and your answer under part 2(c)?

(3)

Complete the argument in p. 26 of lecture 1: Explain why the von Neumann-Morgenstern theory means that one cannot at the same time prefer L^2 over L^1 and also L^3 over L^4.