Seminar exercise 6(1), fall 2013

Exercise for seminar no. 6, 12 November

Parts (2)–(4) were distributed earlier.

(1)

This exercise requires the use of Excel or some other software. The use of spreadsheets is not required to take this course, but since most of you know this from before, I have included an exercise of this type. If you don't know Excel, I recommend that you not try to do this yourself.

The purpose is to do a Monte Carlo simulation along the lines shown at the end of lecture 11. That Excel file is made available in Fronter (the seminar room), as montecarlo.xlsx. If you open it with a Norwegian version of Excel, you should be aware that typically this will use comma as decimal sign (one half is 0,5), and will therefore use semicolon (instead of comma) as separator in a list of arguments. If you want to see all formulas instead of values (the way the spreadsheet was presented in the lecture), you obtain this by clicking Show formulas in the Formulas tab.

You should be aware that each time Excel recalculates cells in the spreadsheet, it will also enter new random numbers (in cells B3:B102 the way the spreadsheet is set up). (Recalculations happen if you press <F9>, or if you enter a new formula.) If you want to avoid this, and use the same sample all the time, you may change the =RAND() in the cells in column B into values, i.e., numbers. This can be done with the Paste special command. However, this is not necessary for estimates to be valid. It may be a good idea to reuse the sample if you want to compare different Monte Carlo estimators.

Assume that $S_0 = 10, K = 8, T = 2, r = 0.05, \sigma = 0.2$.

(1)(a)

From the spreadsheet provided, recalculate ten times and write down the ten estimates of the option values for those recalculations. Then extend the columns to one thousand numbers instead of one hundred. (Each average in row 2 should now use cell 3 to cell 1002 for that column.) Recalculate ten times and write down the ten estimates of the option values for those recalculations. What do you observe about the variation in the estimates if you compare those with samples of 100 and those with samples of 1000?

(1)(b)

Use another part of the spreadsheet to calculate the exact Black-Scholes-Merton formula, equation (14.20) p. 313 in Hull (eighth ed.). (You have to figure out yourself how to do this.) Compare the number you find with the estimates from Monte Carlo. Are they similar?

An application where Monte Carlo simulation is useful

For the call option, we have an analytical formula (see part (b) above), so Monte Carlo simulation was not necessary. You may, however, use Monte Carlo simulation to find the values at time 0 of a claim to a more complicated, non-linear function of S_T to be received at a later date T. If the claim can be written as a known combination of call and put options and the underlying asset itself (see, e.g., the various strategies in ch. 11 in Hull), you could find the separate values of these (options and forward contracts) and add them up. In that case Monte Carlo is not needed. But when the functions are not piecewise-linear, or functions of more than one S_t, Monte Carlo is the easiest method.

Below you are asked to do this for a function of S_1 and S_2, in this case related to taxation of a firm.[1] Consider a firm that produces a quantity Q_t of an investment asset (so you will not have to worry about the valuation at time 0 of this) to be sold at the price S_t in period t. (In part (c), we look at $t = 1$ only. In part (d), we look at $t = 1$ and $t = 2$. We assume that there is no more output after the period(s) we consider. We assume that all investment, production, and tax payment happen once per year, e.g, at the end of each year.) In period 0 the firm invests I. The firm is subject to corporate income taxation with depreciation deductions $a_t \cdot I$, and either no loss offset or imperfect loss offset, to be explained below. Use the values for S_0, r, σ given in the introduction. In parts (c) and (d), you may use the Black and Scholes formula or Monte Carlo simulation. In (e), the Black and Scholes formula cannot be used.

(1)(c)

Assume that the firm (possibly) pays a tax in period 1 at the rate τ. The tax base is $S_1Q_1 - I$, i.e., the depreciation deduction allowed in period 1 is 100 percent, $a_1 = 1$. If the tax base is negative, no tax is paid, there is no refund, and no possibility to carry the deduction forward or backward in time. (This is called "no loss offset.") Show that the

[1] There are several features of this tax system that may lead to a negative after-tax value even when the before-tax value is positive. Compared with exercise 1(d) of seminar 4, the present value of the depreciation deductions is not maintained here, since only the nominal sum of deductions is equal to the investment, not the present value. But worse, the lack of loss offset in (c) and (d) means that the firm is faced with bad outcomes as if there were no tax, but is taxed on the good outcomes. The third problem is that the "partial loss offset" in (e) is insufficient, partly because there is no further deduction (or refund) possibilities after period 2, and partly because the present value of the carryforward is not maintained through interest accumulation. For more on this, see D. Lund, "Petroleum Taxation under Uncertainty—Contingent Claims Analysis with an Application to Norway," *Energy Economics* 14, 1992, 23–31.

tax has a cash flow similar to an option, even though there is in fact no choice whether to exercise or not.[2] Is this a call or put option? Who has the "option," the firm or the authorities (and who will therefore gain from higher uncertainty)? Assume that $I = 16, Q_1 = 2$, and calculate the net value of the firm before tax (as if $\tau = 0$) and under $\tau = 0.28$. What is the average tax rate (the relative reduction in net value due to the tax system)?

(1)(d)

Assume instead that there is production both in period 1 and 2, with $Q_1 = 1$ and $Q_2 = 1$, and $a_1 = a_2 = 0.5$. The tax bases are now $S_1Q_1 - a_1I$ and $S_2Q_2 - a_2I$, and there is again no loss offset. Calculate the net value of the firm before tax and under $\tau = 0.28$. What is the average tax rate?

(1)(e)

Assume that the situation is as in part (d), except that a loss carryforward is allowed in case the tax base in period 1 is negative. This means that the absolute value of this "loss," which amounts to $|S_1Q_1 - a_1I| = a_1I - S_1Q_1$, in that case can be deducted in period 2. There is still no loss offset in period 2. (This means that in case the tax base in period 2 is negative, there is no tax payment in period 2, but also no refund. The tax base in period 2 is either $S_2Q_2 - a_2I$, if there has been no actual loss carryforward from period 1, or $S_2Q_2 - a_2I - (a_1I - S_1Q_1)$ in case there is such a carryforward.)

Use Monte Carlo simulation to calculate the net value of the firm after tax in this situation, and compare with the result from (1)(d). (Hint: Equation (14.7) in Hull.)

[2] For more on this, see M. Samis, G.A. Davis, D. Laughton, and R. Polin, "Valuing uncertain asset cash flows when there are no options: A real options approach," *Resources Policy* 30, 2006, 285–298.