UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4510 – Finance Theory**

Date of exam: Thursday, Nov. 26, 2009   **Grades will be given: Friday, December 18**

Time for exam: 9:00 a.m. – 12:00 noon

The problem set covers 3 pages (incl. cover sheet)

Resources allowed:
- All written and printed resources, as well as calculator, is allowed.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Please do not forget the periodic course evaluation for ECON4510, which you will find on the website for the course. The deadline is December 11!

The exam consists of three problems. They count equally. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

Problem 1

Consider a risk-averse person who maximizes expected utility. Show that in a simple portfolio problem with one risky and one risk free asset, the person will invest a strictly positive amount in the risky asset if and only if its expected rate of return, $E(\tilde{r})$, exceeds the rate of return on the risk free asset, r_f. Give a verbal interpretation of the result.

Problem 2

Consider an economy where the standard Capital Asset Pricing Model holds, with a large number of different shares traded in the stock market. You are not asked to derive the model or even state all assumptions behind it. Consider two of the many firms in this economy, and assume that they do not pay any dividends in the period we are concerned with. For simplicity we assume they are financed by equity only, no debt. The total value today of all shares in the two firms are X_0 and Y_0, respectively, and their respective total share values one period into the future are $\tilde{X}_1$ and $\tilde{Y}_1$, both stochastic. The ratio of the total values today is $X_0/Y_0 = 1/5$.

(a)

A merger between the firms is considered. Assume that the merged firm will have total value next period equal to $\tilde{Z}_1 = \tilde{X}_1 + \tilde{Y}_1$. Show how today's value of the merged firm, Z_0, relates to X_0 and Y_0. Show how the beta of the shares in the merged firm, β_Z, relates to the betas of the two existing firms, β_X and β_Y.

(b)

The rates of return of the shares of the firms are $\tilde{r}_X$ and $\tilde{r}_Y$, respectively, with the properties that $E(\tilde{r}_X) = 0.04$, $E(\tilde{r}_Y) = 0.16$, $\mathrm{var}(\tilde{r}_X) = 0.09 = 0.3^2$,

$\text{var}(\tilde{r}_Y) = 0.16 = 0.4^2$, and $\text{cov}(\tilde{r}_X, \tilde{r}_Y) = 0.02$. Assuming that the merger does not happen: Show that the minimal variance of the rate of return of any possible portfolio of these two shares is $0.6/9 \approx 0.06667 \approx 0.2582^2$. Illustrate with a suitable diagram which portfolios can be created from the two shares. Assuming instead that the merger happens, show the location of the merged firm's shares in the diagram.

(c)

Discussing the possible benefits and drawbacks of the merger, one person argues, "The return on the shares of the merged firm will have a lower variance than the shares of any of the two existing firms. This is a benefit for shareholders." Discuss both parts of this statement: What can you say about the first, factual claim? What can you say about the benefit for shareholders?

Problem 3

(a)

Show how to derive the value of a call option by an absence-of-arbitrage argument in a binomial model of share prices. Assume that the option is of European type with expiration one period into the future, and that the share does not pay dividends during that period. In this model the relative change in the share price between two periods is either u or d, while the riskless interest rate is r.

(b)

A variable defined as

$$p = \frac{e^r - d}{u - d}$$

has a particular meaning. Does this variable relate to the call option value, and if yes, how? Does this variable play a role in the absence-of-arbitrage argument, and if yes, how? Does this variable relate to the probability of some change in the share price, and if yes, how?