UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4510 – Finance Theory**

Date of exam: Tuesday, May 24, 2011 **Grades are given: Tuesday June 14, 2011**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 4 pages (incl. cover sheet)

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of three problems. They count equally. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

Problem 1

Consider a person who plans for only one future period, $t = 1$. After consumption today, $t = 0$, has been taken care of, the person has wealth $W > 0$ to invest to provide for consumption, $\tilde{C}$, at $t = 1$. The budget for $\tilde{C}$ consists only of the results from the investment of this wealth. The person has logarithmic utility, i.e., the person maximizes $E[u(\tilde{C})] \equiv E[\ln(\tilde{C})]$. There are only two investment opportunities. A bank offers risk free borrowing and saving at an interest rate r_f. There is also a risky asset with a rate of return $\tilde{r}$.

(a)

Discuss whether we should assume that $\Pr(\tilde{C} > 0) = 1$, i.e., that the consumption at $t = 1$ will be a positive number for sure.

(b)

Assume that $\tilde{r}$ has only two possible outcomes, r_1 and r_2, with $\Pr(\tilde{r} = r_1) = p$. Show that under some assumptions, the optimal amount to invest in the risky asset is

$$a^* = \frac{W(1 + r_f)[E(\tilde{r}) - r_f]}{(r_1 - r_f)(r_f - r_2)}.$$

(c)

Give a verbal interpretation of the solution for a^*. You should at least address the following questions: Could the equation give a negative value? What would be the economic interpretation of this? How does uncertainty about $\tilde{r}$ influence the optimal risky investment?

Problem 2

You are asked to sketch two different opportunity sets in the same diagram, and then to consider what can be said about which of those opportunity sets an investor would prefer. All you know about investors is that they are risk averse and their preferences only depend on mean and variance.

Consider a pair of risky securities, 1 and 2, with rates of return $\tilde{r}_j, j = 1, 2$, with the properties $\sigma_1^2 = \text{var}(\tilde{r}_1) = 0.2^2 = 0.04$, $\sigma_2^2 = \text{var}(\tilde{r}_2) = 0.4^2 = 0.16$, and $\sigma_{12} = \text{cov}(\tilde{r}_1, \tilde{r}_2) = 0.01$. Two additional pieces of information are needed in order to define the opportunity set of portfolios from the two securities: Short selling may or may not be allowed (see below). With regard to the expected rates of return, $\mu_1 = E(\tilde{r}_1)$ and $\mu_2 = E(\tilde{r}_2)$, you are asked to consider two cases:

Case (i): $\mu_1 = 0.06$, $\mu_2 = 0.18$.

Case (ii): $\mu_1 = 0.14$, $\mu_2 = 0.02$.

(a)

For case (i), find that portfolio of the two securities which has a rate of return with the lowest variance. Show that this variance is $0.035 \approx 0.187^2$. Explain why the same result holds for case (ii). Sketch the opportunity sets for the two cases in the same diagram.

(b)

Can you determine whether an investor would prefer case (i) to case (ii) when short selling is not allowed?

(c)

Can you determine whether an investor would prefer case (i) to case (ii) when short selling is allowed?

Problem 3

Consider a call option (American or European) with expiration at date T on a share which for sure does not pay dividends in the time interval $(0, T)$.

The price at time 0 of the share is S, and the option's exercise price is K. The interest rate on riskless bonds is r.

(a)

Explain why we can assume that the option value is greater than or equal to $S - Ke^{-rt}$.

(b)

Explain how the result in (a) can be used to show that the value of an American call option of the type described has the same value as a call option that is European, but otherwise identical.

(c)

Explain why the explanations above do not hold if there is a possibility that the stock pays dividends at some specific dates between time 0 and T. Explain why it is not optimal to exercise the American option for most of the time between 0 and T.