UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam: **ECON4510 – Finance Theory**

Date of exam: Wednesday, June 8, 2011

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 4 pages (incl. cover sheet)

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of three problems. They count equally. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

Problem 1

Consider an investment project which at time $t = 1$ can produce one unit of output that will be sold at a price $\tilde{P}$ that is uncertain as seen from time $t = 0$. The investor is not able to affect the probability distribution of $\tilde{P}$. Starting the project requires an investment I (an exogenously given magnitude) at $t = 0$. There are no other costs or revenues.

(a)

Assume that this project may be undertaken in an economy in which the Capital Asset Pricing Model holds. The only choice is to start the project or not. What is the value of the project according to this model? What is the criterion for starting the project according to the model?

(b)

Assume now that there are many identical potential projects. What does the model say about the optimal number of projects to start? Explain!

(c)

How does the value of a project (from part (a)) depend on

- The risk free interest rate, r_f?
- The variance of $\tilde{P}$, $\sigma_P^2 = \text{var}(\tilde{P})$?
- The risk aversion of the investor?

How does the value relate to the rate of return on the market portfolio, $\tilde{r}_m$? Give an verbal interpretation of your answers to these four questions.

(d)

Assume that a firm invests in one project. For simplicity, assume that the firm has no debt and pays no taxes. Immediately after investment, the shares in the firm are traded in the stock market. What will be the beta value of these shares, expressed in terms of the same variables you used in part (a)? How does this depend on the magnitude of I? Explain!

Problem 2

As in Problem 1, consider an investment project which at time $t = 1$ can produce one unit of output that will be sold at a price $\tilde{P}$ that is uncertain as seen from time $t = 0$. The investor is not able to affect the probability distribution of $\tilde{P}$. Starting the project requires an investment I (an exogenously given magnitude) at $t = 0$. There are no other costs or revenues.

(a)

Consider an investor who maximizes a utility function $U = m - as^2$, where $\tilde{C}$ is consumption at $t = 1$, $m = E(\tilde{C})$, and $s^2 = \text{var}(\tilde{C})$. The investor has a given wealth W at $t = 0$. This can be transformed into consumption at $t = 1$ in two ways. All or a part of W may be deposited in a bank, and all or a part of W may be invested in a number, n, of identical projects of the type described. A bank deposit is risk free and yields the interest rate r_f.

Show that if n projects are undertaken, utility will be

$$(W - nI)(1 + r_f) + n\bar{P} - an^2\sigma_P^2,$$

where $\bar{P} = E(\tilde{P})$.

(b)

Under what condition will the investor prefer to start one project, compared to depositing all of W in the bank? Compare with the situation in Problem 1(a), and give a verbal interpretation of the difference.

(c)

If you treat n as a continuous variable, is there an optimal number of projects to start for an investor, in the situation described? Compare with the answer in Problem 1(b), and give a verbal interpretation of your answers.

Problem 3

Consider the well-known function

$$c(S_0, K, r, \sigma, T) = S_0 N(d_1) - Ke^{-rT} N(d_2),$$

where $N(\cdot)$ is the standard normal cumulative distribution function, and the variables d_1 and d_2 are defined by

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}},$$

and

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

(a)

Explain how the variables are defined, and what the function is meant to give us. In this connection, give a verbal explanation of why c is a decreasing function of K.

(b)

Explain how the function, under some conditions, may be useful to find a forward-looking estimate of σ.

(c)

Explain how the function, under some conditions, may be useful to find the value of a put option.