UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4510 – Finance Theory, spring 2012**

Date of exam: Tuesday, May 22, 2012 **Grades will be given: June 12, 2012**

Time for exam: 2:30 p. m. – 5:30 p.m.

The problem set covers 5 pages (incl. cover sheet)

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of three problems. They count equally. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

You are warned that there may be information given which is not necessary to answer the problems.

Problem 1

(a)

Explain what is meant by *risk aversion* and by the *measure of absolute risk aversion.*

(b)

Consider the equation

$$E\left[U(Y+\tilde{Z})\right] = U\left(Y + E(\tilde{Z}) - \Pi(Y,\tilde{Z})\right).$$

Define the variables in the equation and explain in particular what is the meaning of $\Pi(Y,\tilde{Z})$. Give a verbal explanation of why and how U and Y may affect $\Pi(Y,\tilde{Z})$.

(c)

Under some assumption(s) there is a particular relation between the measure of absolute risk aversion and $\Pi(Y,\tilde{Z})$. State (i.e., write down) and explain this relation, the assumption(s) behind it, and explain what other variable(s) appear(s) in it.

(d)

Show mathematically how to derive the relation mentioned in (c).

Problem 2

Consider an economy where the assumptions behind the standard Capital Asset Pricing Model hold for the period between time zero and time one. There are many assets and many agents, but no taxes. The risk free interest rate is r_f, and the rate of return of the market portfolio is $\tilde{r}_m$, with expectation $\bar{r}_m = E(\tilde{r}_m) > r_f$ and variance $\sigma_m^2 = \text{var}(\tilde{r}_m) > 0$. Consider a corporation that will have a cash flow $\tilde{X}$ at time 1. The corporation has no debt. The cash flow has expectation $\bar{X} = E(\tilde{X})$, variance $\sigma_X^2 = \text{var}(\tilde{X}) > 0$, and covariance $\sigma_{Xm} = \text{cov}(\tilde{X}, \tilde{r}_m) > 0$ with the rate of return on the market portfolio.

(a)

Write down a formula for the value of the shares in the corporation at time zero as a function of some or all of the parameters mentioned in the introduction. Give a verbal interpretation of how the value is affected by the riskiness of $\tilde{X}$.

(b)

Define the beta value of the shares in the corporation. How is this beta value affected by an increase in σ_X^2 (assuming that the increase is perceived at time zero)? How is the beta value affected by an increase in σ_{Xm} (likewise, perceived at time zero)? You may analyze each of these changes as if the other variable is unchanged, i.e., first, σ_X^2 is increased while σ_{Xm} is unchanged, then vice versa, that σ_{Xm} is increased while σ_X^2 is unchanged.

(c)

The CAPM is often illustrated in a $(\sigma, E(\tilde{r}))$ diagram with a curve known as the frontier portfolio set and a line (or a ray, to be precise) known as the capital market line. Assume that the beta of the shares is around 1.2. Sketch the typical location of the corporation in the diagram before the increases mentioned in part (b). Then show the effects on this location of each of these two increases, separately.

Problem 3

Assume that the binomial share and option model is valid. Consider a share which for sure does not pay any dividend in the periods we focus on. All agents know that if the share price at time t is S_t, then the share price at $t+1$ will be uS_t or dS_t. Consider a European put option on this share with two periods left to maturity, when the share price today (time zero) is $S_0 = 10.00$, the exercise price of the option is $K = 8.82$, the one-period interest rate factor is $e^r = 1.1$, the factor u is 1.3, and the factor d is 0.8. The probability of $(S_{t+1}/S_t = u)$ is $p^* = 0.8$.

Since calculators are not allowed for this exam, there is a table at the end which provides some calculations that may be useful for answering the questions that follow. The table also contains some additional calculations which are not needed, in order that the correct methods for answering the questions shall not be too obvious.

(a)

Find the values of the option at the maturity date for the different possible outcomes of S_2. Show that the market value of the option at time zero, using the risk-neutral valuation model, is 0.32.

(b)

Verify that the market value of the option at time zero, based on a replicating portfolio strategy, is also 0.32.

(c)

Assume that the option is instead traded at the price $p_0 = 0.42$ at time zero. Show that this creates an arbitrage opportunity. Show how to take advantage of the opportunity: What should be bought and sold at time zero? For each of the possible observations of the share price at time one, what should be bought and sold at that time?

Table of calculations; some of these may be useful

$0.605 \cdot 13$	$=$	7.865	$2.42/4$	$=$	0.605
$7.865/1.1$	$=$	7.15	$0.6 \cdot 0.6$	$=$	0.36
$6.292/1.1$	$=$	5.72	$0.6 \cdot 0.4$	$=$	0.24
$0.605 \cdot 8$	$=$	4.84	$0.42 \cdot 0.42$	$=$	0.1764
$13 \cdot 0.16$	$=$	2.08	$0.88/5$	$=$	0.176
$2.42/1.21$	$=$	2	$0.4 \cdot 0.4$	$=$	0.16
$1.3 \cdot 1.3$	$=$	1.69	$0.8 \cdot 0.2$	$=$	0.16
$1.1 \cdot 1.1$	$=$	1.21	$0.176/1.1$	$=$	0.16
$1.3 \cdot 0.8$	$=$	1.04	$0.32 \cdot 0.32$	$=$	0.1024
$0.1764/0.24$	$=$	0.735	$0.32 \cdot 0.176$	$=$	0.05632
$0.8 \cdot 0.8$	$=$	0.64	$0.2 \cdot 0.2$	$=$	0.04