# *UNIVERSITY OF OSLO*
# *DEPARTMENT OF ECONOMICS*

**Exam: ECON4510 – Finance Theory**

Date of exam: Thursday, November 28, 2013 **Grades are given: December 20, 2013**

Time for exam: 2.30 p.m. – 5.30 p.m.

The problem set covers 5 pages

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of three problems. They count equally. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

You are warned that there may be information given which is not necessary to answer the problems.

# Problem 1

## (a)

Consider the function $U(C) = aC^b$, with $a$ and $b$ being constant, real numbers. The function is only defined for $C \geq 0$. What conditions must be satisfied for $E[U(C)]$ to represent the utility of a von Neumann-Morgenstern type person who is risk averse? What are the measures of absolute and relative risk aversion for this utility function?

## (b)

Assume that the person with the utility function mentioned in part (a) is planning for only one future period, $t = 1$. The person has wealth $W > 0$ to invest to provide for future consumption, $\tilde{C}$, at $t = 1$. The budget for $\tilde{C}$ consists only of the results from the investment of this wealth. There are only two investment opportunities. A bank offers risk free borrowing and saving at an interest rate $r_f$. There is also a risky asset with a rate of return $\tilde{r}$. Formulate the person's maximization problem for the future period, and write down the first-order condition for a maximum.

## (c)

Assume that $\tilde{r}$ has only two possible outcomes, $r_1$ and $r_2$, with numbers chosen such that $r_1 > r_2$, and with $\Pr(\tilde{r} = r_1) = p$. Show that under some assumptions, the optimal amount to invest in the risky asset is

$$v^* = \frac{W(1 + r_f)(X - 1)}{r_1 - r_f + X(r_f - r_2)},$$

where $X$ is defined by

$$X = \left[ \frac{p(r_1 - r_f)}{(1-p)(r_f - r_2)} \right]^{\frac{1}{1-b}}.$$

One assumption you will need is $r_2 < r_f < r_1$. Give an economic interpretation of this assumption.

### (d)

Based on the assumptions in (c), show that if $E(\tilde{r}) = r_f$, then $v^* = 0$. With this situation as a starting point, what would be the effect on $v^*$ of an increase in $p$? Give an economic interpretation of this effect.

### (e)

Based on the assumptions in (c), and with $v^* > 0$ as a starting point, what is the effect on $v^*$ of an increase in the individual's risk aversion? Give an economic interpretation of this effect.

## Problem 2

### (a)

Consider a firm that can make a real investment $I$ at time $t = 0$. If undertaken, the investment starts a project, which will have a cash flow at time $t = 1$ equal to $PQ$. Here, $P$ is a product price in a competitive market, uncertain as seen from $t = 0$, while $Q$ is a quantity, known with certainty from $t = 0$. In part (a), the firm has no other activity. Discuss what is the value of the firm, based on the Capital Asset Pricing Model. You can disregard time periods after $t = 1$. You can assume that the firm does not borrow and that there are no taxes to be paid. Please remember to give a short definition of each variable you introduce.

### (b)

Assume now that the firm discovers another investment opportunity, with investment cost $J$ at $t = 0$ and cash flow $ZY$ at $t = 1$. Again, $Z$ is an uncertain price and $Y$ is a certain quantity, if the investment is undertaken.

What is the value of the firm when it has both opportunities? Derive a formula and discuss whether it depends on $P$ and $Z$ being stochastically independent.

### (c)

How will the beta of shares in the firm in part (b) (call this $\beta_b$) relate to the beta of the shares in the firm in part (a) (call this $\beta_a$) and the beta of the shares in a (hypothetical) firm with only the second of the two investment opportunities (call this $\beta_c$)? How does $\beta_b$ depend on $I$ and $J$?

## Problem 3

Since calculators are not allowed for this exam, there is a table at the end which provides some calculations that may or may not be useful for answering the questions that follow. In order not to make the correct answers too obvious, some useless numbers are included in the table.

### (a)

Assume that the binomial share and option model is valid: Consider a share which for sure does not pay any dividend in the periods we focus on. All agents know that if the share price at time $t$ is $S_t$, then the share prices at $t+1$ will be $uS_t$ or $dS_t$. Consider a European call option with two periods left to maturity, when the share price today is $S_0 = 10$, the exercise price of the option is $K = 8$, the one-period interest rate factor is $e^r = 1.1$, the factor $u$ is 1.3, and the factor $d$ is 0.8. The probability of an upward movement is $\Pr(S_t/S_{t-1} = u) = 0.8$. Find the value of the call option at time $t = 0$.

### (b)

Another call option on the same share, with expiration at $t = 2$, has a (theoretical) value at $t = 0$ equal to 1.8. What is the exercise price of this option?

**Table of calculations; some of these may be useful**

| | | | | | |
|---|---|---|---|---|---|
| $8.898/0.84$ | $\approx$ | $10.5929$ | $0.48 \cdot 2.4$ | $=$ | $1.152$ |
| $0.36 \cdot 16.9$ | $=$ | $6.084$ | $0.16 \cdot 6.4$ | $=$ | $1.024$ |
| $0.48 \cdot 10.4$ | $=$ | $4.992$ | $1.092/1.21$ | $\approx$ | $0.9025$ |
| $6.016/1.21$ | $\approx$ | $4.9719$ | $0.6 \cdot 0.6$ | $=$ | $0.36$ |
| $3.6 \cdot 1.21$ | $=$ | $4.356$ | $0.55 \cdot 0.55$ | $=$ | $0.3025$ |
| $2.814/0.84$ | $=$ | $3.35$ | $0.55 \cdot 0.45$ | $=$ | $0.2475$ |
| $0.36 \cdot 8.9$ | $=$ | $3.204$ | $0.6 \cdot 0.4$ | $=$ | $0.24$ |
| $1.8 \cdot 1.21$ | $=$ | $2.178$ | $0.45 \cdot 0.45$ | $=$ | $0.2025$ |
| $1.1 \cdot 1.1$ | $=$ | $1.21$ | $0.4 \cdot 0.4$ | $=$ | $0.16$ |