UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4510 – Finance theory**

Date of exam: Wednesday, May 20, 2015 **Grades are given: June 9, 2015**

Time for exam: 2.30 p.m. – 5.30 p.m.

The problem set covers 5 pages (incl. cover sheet)

Resources allowed:

- No resources allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences)

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of three problems. They count equally. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

You are warned that there may be information given which is not necessary to answer the problems.

Since calculators are not allowed for this exam, there is a table at the end which provides some calculations that may or may not be useful for answering the questions that follow. In order not to make the correct answers too obvious, some useless numbers are included in the table.

Problem 1

(a)

Consider the function $U(W) = cW^2 + bW + a$, with a, b and c being constant, real numbers. You can assume that for our purpose, the function is defined only for $W \geq 0$. Discuss under what conditions this function can be a meaningful utility function for someone who is risk averse and maximizes expected utility. Sketch the graph of the function. What is the measure of absolute risk aversion, $R_A(W)$, for this utility function? Is $R_A(W)$ constant, increasing, and/or decreasing? What can you say about the advantage(s) and drawback(s) of using this function as a utility function to describe risk averse maximization of expected utility?

(b)

Assume now that conditions are satisfied so that the function defined in (a) can be meaningfully used. Show that a person with this utility function, planning for one future period with risky wealth, will care only about the mean and variance of wealth, $E(W)$ and $\text{var}(W)$.

(c)

For this part, you can rely on the following property of a uniform distribution: If W is uniformly distributed on the interval $[0, 6]$, then $E(W) = \text{var}(W) = 3$.

Discuss the following statement: "Any person with the quadratic utility function defined in (a) will be indifferent between the following two alternatives: (i) a risky future wealth that is lognormally distributed with $E(W) = 3$ and $\text{var}(W) = 3$, and (ii) a risky future wealth that is uniformly distributed on the interval $[0, 6]$."

(d)

Discuss the following statement: "Any person with the quadratic utility function defined in (a) will be indifferent between the following two alternatives: (i) a risky future wealth that is lognormally distributed with $E(W) = 3$ and $\text{var}(W) = 3$, and (ii) a risky future wealth that is normally distributed with $E(W) = 3$ and $\text{var}(W) = 3$."

Problem 2

Consider an economy described by the standard Capital Asset Pricing Model (CAPM) between two points in time, $t = 0$ and $t = 1$. Consider a potential investment I at $t = 0$ to start a firm to produce a known homogeneous output Q to be sold an uncertain price $\tilde{P}$ at $t = 1$. You can assume that immediately after the investment, shares in the firm will be traded in the stock market, and, for simplicity, that the firm does not pay taxes.

(a)

Show how the CAPM β of the shares of the firm, as described in the introduction, will be related to the covariance between the output price and the rate of return on the market portfolio, $\tilde{r}_m$. What will be the relation between β and the amount of real investment, I? Give a brief verbal interpretation.

(b)

Assume now that a fraction $(1 - a) \in (0, 1)$ of the investment, I, will be financed by debt. Assume for simplicity that the debt can and will be paid back with interest with full certainty at $t = 1$. Under these assumptions, explain how the decision by the original investors, whether to invest or not, is related to the fraction $(1 - a)$ or a. Give a brief verbal interpretation.

(c)

Explain how the β of shares in the firm, when it has the obligation to pay back debt with interest as in part (b), will be related to the the fraction $(1-a)$ or a. Give a brief verbal interpretation.

Problem 3

(a)

Explain in words why a put option value is decreasing in the value of the underlying asset, and why the value of an American put option is greater than or equal to the value of an otherwise identical European put option. Can you prove this second statement by absence of arbitrage?

(b)

Consider a put option with expiration two periods into the future ($T = 2$), on a stock which for sure does not pay dividends between now (time zero) and then (T). The stock price now is $S_0 = 100$. The exercise price of the put option is $K = 105$. It is known that during each period, there is a probability $p^* = 0.9$ that the stock will increase in value by 10 percent, and a probability $1 - p^*$ that the stock will decrease in value by 10 percent. There exist risk free bonds with continuous compounding such that a risk free interest rate r implies an interest factor $e^r = 1.05$ per period.

What is the value of the put option at time zero in case it is a European option?

(c)

What is the value at time zero of the put option mentioned in part (b) in case it is an American option?

Table of calculations; some of these may be useful

$0.9 \cdot 16$	$=$	14.4	$4.8214/1.05$	$\approx$	4.5918
$14.4/1.05$	$\approx$	13.7143	$3.5714/1.05$	$\approx$	3.4014
$0.9 \cdot 13.7143$	$\approx$	12.3429	$2.0143/1.05$	$\approx$	1.9184
$12.3429/1.05$	$\approx$	11.7551	$1.5/1.05$	$\approx$	1.4286
$12/1.05$	$\approx$	11.4286	$1.2571/1.05$	$\approx$	1.1973
$0.75 \cdot 11.4286$	$\approx$	8.5714	$1.4286 \cdot 0.75$	$\approx$	1.0714
$8.5714/1.05$	$\approx$	8.1633	$0.6/1.05$	$\approx$	0.5714
$7.8/1.05$	$\approx$	7.4286	$0.9 \cdot 0.5714$	$\approx$	0.5143