UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4510 – Finance Theory**

Date of exam: Thursday, May 26, 2016 **Grades are given: June 16, 2016**

Time for exam: 2.30 p.m. – 5.30 p.m.

The problem set covers 4 pages (incl. cover sheet)

Resources allowed:
- No written or printed resources – or calculator - is allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences)

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of three problems. They count equally. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

Problem 1

(a)

Explain what is meant by riskless arbitrage, and why we assume such arbitrage is incompatible with a market equilibrium.

(b)

Use the concepts from part (a) to prove that under some conditions, a forward price is determined by the equation $F_0 = S_0 e^{rT}$, where S_0 is the price of some asset. Be careful to explain the meaning of the variables and the conditions needed for your proof to be valid.

(c)

Consider a forward contract that satisfied the equation from (b) when it was entered into. What will be the value of (a long position in) the contract at some later point in time? Explain!

(d)

If it is revealed that the asset with price S_0, mentioned in part (b), will give a payout (e.g., a dividend) before the delivery date, what will be the effect on the forward price, if any? You can assume that before this was revealed, the market assumed that there would be no payout.

Problem 2

Consider the well-known function

$$c(S_0, K, r, \sigma, T) = S_0 N(d_1) - K e^{-rT} N(d_2),$$

where $N(\cdot)$ is the standard normal cumulative distribution function, and the variables d_1 and d_2 are defined by

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}},$$

and

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

(a)

Explain how the five arguments of the c function are defined, and what the function is meant to give us. Explain the main conditions needed for the formula to be valid. You are not asked to give a mathematical proof that the formula holds under those conditions.

(b)

Explain how the function, under some conditions, may be useful to find a forward-looking estimate of σ.

(c)

When S_0 becomes very large, and the other variables are fixed, the c function approaches the value described in part (c) of Problem 1 above. Show this mathematically. Give a verbal interpretation.

Problem 3

Consider an economy described by the standard Capital Asset Pricing Model (CAPM) between two points in time, $t = 0$ and $t = 1$. Consider a potential investment I at $t = 0$ to produce a known homogeneous output Q to be sold an uncertain price $\tilde{P}$ at $t = 1$. No other factor input is necessary. The output quantity is determined with full certainty by the amount invested through the function $Q = f(I) = aI^b$, where a and b are positive constants. The valuation at $t = 0$ of a claim to one unit of output at $t = 1$ is denoted $V(\tilde{P})$, and is assumed to be strictly positive. For simplicity, assume that the firm has no other activity. In part (a) and (b) the firm does not pay taxes.

(a)

What is the first-order condition for choice of an optimal amount to invest? Explain why we may want to assume $b < 1$. Explain how and why the optimal choice does, or does not, depend on the risk aversion of the owners of the firm.

(b)

Show that the first-order condition leads to $I/Q = bV(\tilde{P})$.

(c)

Show how the CAPM β of the shares of the firm, immediately after investment, will be related to the covariance (assumed to be positive) between the output price and the rate of return on the market portfolio, $\tilde{r}_m$. Will there be a relation between β and the parameter b of the production function? Give a brief verbal interpretation.

(d)

Assume now that the firm pays a tax in $t = 1$ (but not at $t = 0$). The tax payment is $\tau \cdot (\tilde{P}Q - cI)$, where $\tau \in (0, 1)$ is a tax rate, and $c = 1 + r_f$. The cash flow to the firm at $t = 1$ is thus $\tilde{P}Q(1 - \tau) + \tau cI$. For simplicity, assume that the firm will earn the tax value of the deduction, τcI, at $t = 1$ even when $\tilde{P}$ has low outcomes, so that τcI can be considered as a risk free cash flow.

Show in this case how the CAPM β of the shares of the firm, immediately after investment, will be related to the covariance between the output price and the rate of return on the market portfolio, $\tilde{r}_m$. Will there in this case be a relation between β and the parameter b of the production function? Give a brief verbal interpretation.