i Candidate instructions

ECON4510 - Finance Theory

This is some important information about the written exam in ECON4510. Please read this carefully before you start answering the exam.

Date of exam: Monday, May 13, 2019
Time for exam: 09.00 - 12.00 (3 hours)
The problem set: The problem set consists of 3 problems. They will be given weight as indicated.

Some advice: Start by reading through the whole exam, and make sure that you allocate time to answering problemss you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve. It is better to try to do something on each problem than to get bogged down with one problem. If you find you are spending too much time on one problem, stop working on it and plan to get back to it if you have time at the end. Make sure you state any assumptions you make.

Sketches: You may use sketches on all questions. You are to use the sketching sheets handed to you. You can use more than one sketching sheet per question. See instructions for filling out sketching sheets at the bottom. It is very important that you make sure to allocate time to fill in the headings (the code for each problem, candidate number, course code, date etc.) on the sheets that you will use to add to your answer. You will find the code for each problem under the problem text. You will NOT be given extra time to fill out the "general information" on the sketching.

Access: You will not have access to your exam right after submission. The reason is that the sketches with equations and graphs must be scanned in to your exam. You will get access to your exam within 2-3 days.

Resources allowed: No written or printed resources - or calculator - is allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences).

Grading: The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Grades are given: Monday, June 3, 2019

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Problem 1 - 35%

Assume that all investors have linear-quadratic preferences and suppose they can invest their wealth in a set of risky assets. Moreover, the returns on the assets are imperfectly correlated and the expected return and variance differs across assets.

(A) [5%] Suppose there is no risk free asset. Illustrate, using a diagram, the set of portfolios that investors will choose in equilibrium.

(B) [5%] Explain how the optimal portfolios change when a risk free asset is introduced

(C) [10%] Suppose a risky asset i has an expected return equal to the risk free rate $(E(r_i) = r_f)$ and the same standard deviation as the return on the market portfolio.

i. Explain why all individuals want to hold this asset in equilibrium even though it offers a low equilibrium return.

ii. What must the covariance be between the rate of return on the asset, r_i, and the return on the market portfolio in equilibrium?

(D) [15%] Empirical testing of CAPM:

i. Explain how CAPM can be tested empirically.

ii. Review some empirical evidence that CAPM should be rejected.

iii. How could a die-hard CAPM believer hold on to CAPM despite the evidence above?

Fill in your answer here and/or on sketching paper

Maximum marks: 35

Attaching sketches to this question?
Use the following code:

XXXXXXX

2 Problem 2 - 45%

Consider an individual with preferences $U\left(c_1, c_2\right)=u\left(c_1\right)+\beta u\left(c_2\right)$, where the utility function $u(c)$ is constant relative risk aversion (CRRA);
$u\left(c\right)=\frac{c^{1-\gamma}}{1-\gamma}$
where $\gamma \geq 0$. The individual has an initial wealth W and has no other income.

(A) [5%] Show that the parameter γ is the relative risk aversion for this individual.

(B) [10%] Assume that in period 1 the individual can invest in stocks and bonds, where stocks pay a risky return r_s in period 2 and bonds have a safe return r_f in period 2.

i. Write down the optimization problem for the individual.
ii. Show that the first-order conditions for this individual can be expressed as

$$1=E\left\{\frac{\beta u'\left(c_2\right)}{u'\left(c_1\right)}\cdot\left(1+r_f\right)\right\}$$
$$1=E\left\{\frac{\beta u'\left(c_2\right)}{u'\left(c_1\right)}\cdot\left(1+r_s\right)\right\}.$$

(C) [15%] Assume that c_1 and c_2 are aggregate consumption and that all households have the same CRRA utility function U above. Moreover, assume that the aggregate consumption growth, c_{t+1}/c_t, and the risky return $1+r_s$ are jointly log-normal. Show that the equity premium can be expressed as
$E\left(r_s\right)-r_f \approx \gamma \cdot cov\left(\ln\left(\frac{c_{t+1}}{c_t}\right), r_s\right)$

(D) [7%] Give an economic interpretation of the formula for the equity premium implied by the model, i.e., the equation in (2.C) above.

(E) [5%] Mehra and Prescott (1985) documented the following annual statistics for the 1889-1980 period for USA: $E\left(r_s\right)=7\%$, $E\left(r_f\right)=1\%$, and the following covariance matrix:

Variance-covariance matrix. USA 1889-1978

	$1+r_{s,t+1}$	c_{t+1}/c_t	
$1+r_{s,t+1}$	0.0274	0.0022	
c_{t+1}/c_t		0.0013	

Explain why these empirical observations are puzzling in light of the equity premium implied by the model, i.e., the equation in (2.C) above.

(F) [3%] Assume that a new asset i is introduced and that the return on this asset is negatively correlated with consumption growth, i.e., $corr(\frac{c_{t+1}}{c_t}, r_i) < 0$. Will the expected return on this asset be lower or higher than the safe interest rate r_f? Give thorough intuition for your argument.

Fill in your answer here and/or on sketching paper

Maximum marks: 45

Attaching sketches to this question?
Use the following code:

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3 Problem 3 - 20%

Consider a two-period binominal model for an asset. The price of the asset is $S = 1$ in the first period. Assume that the interest rate is zero ($r = 0$). Moreover, between period 1 and 2 the share price of the asset will either increase with 10% (which happens with probability 0.6) or fall with 10% (which happens with probability 0.4). The asset does not pay any dividends. Consider a call option with a strike price $K = 1$ in the second period.

(A) [1%] When (i.e., in which states) should the option be exercised?

(B) [9%] Calculate the replicating portfolio. Explain why the replicating portfolio (for a call option) in the binominal model will always involve leveraged positions, i.e., debt ($B \leq 0$).

(C) [5%] Calculate the value of the call option in the first period.

(D) [5%] Why is the probability of a price increase irrelevant for the price of the option?

Fill in your answer here and/or on sketching paper

Maximum marks: 20

Attaching sketches to this question?
Use the following code:

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