ECON4510 - Spring 2020

1 CAPM

Consider an economy where individuals have linear-quadratic preferences. Individuals are endowed with initial wealth which they invest in a set of risky assets.

1. Assume first that there is no riskfree asset. Explain the set of portfolios that households would choose in equilibrium. You may draw a diagram.

2. Explain how the set of portfolios households will choose, changes when one of the assets is risk free. You may draw a diagram.

3. Suppose the riskfree interest rate is $r_f = 0$. Suppose the market portfolio is the set of all stocks in the world and that this portfolio has an expected return of $E(R_M) = 5\%$ and a standard deviation of 10%. Consider a stock in Telenor. Suppose Telenor has a return R_i with standard deviation $\sigma(R_i) = 30\%$ and a covariance with the market portfolio of $cov(R_i, R_M) = 0.02$.

    (a) What is the expected return $E(R_i)$ for Telenor stocks?

    (b) Note that the variance of Telenor's return is $\sigma^2(R_i) = 0.09$. What is the specific risk (or idiosyncratic risk) and market risk (or systematic risk) for Telenor?

4. Suppose you observe that over time, the covariance with the market portfolio remains $cov(R_i, R_M) = 0.02$ but the average return on Telenor is 15% (and larger than the answer you found in question 3). What would you conclude about the CAPM model?

2   Portfolio choice

Suppose CAPM holds. There are 3 risky assets in the market and one risk free
asset. The variance-covariance matrix of the returns on the risky assets is

Covariance	A	B	C
A	0.0064	0.0036	0.0012
B	0.0036	0.0144	0.0036
C	0.0012	0.0036	0.0225

The mean returns are

	$E\left(R_i\right)$
Asset A	0.10
Asset B	0.20
Asset C	0.30

The market portfolio is $0.1A + 0.4B + 0.5C$, and the risk-free rate is 0.03.

1. Show that the market portfolio has mean return of 0.24 and a stadnard
   deviation of 0.0992 (you can do the rest of the exercises even if you struggle
   with this question).

2. If an investor requires a return of 14%, what is the optimal allocation of
   her wealth among all assets? And what level of risk does she bear?

3. What if the required return is 36%? (Assume that the investor can borrow
   and lend at the risk-free rate.)

4. Calculate the Sharpe Ratio of the market portfolio. Consider another
   portfolio $0.7A+0.1B+0.2C$, what is the expected return of this portfolio?
   Explain why – in equilibrium – the Sharpe ratio of the alternative portfolio
   must be lower than the Sharpe ratio.

5. Compute each asset's beta.

6. Suppose there existed an asset with $\beta = 1.5$. Using the security market
   line, what would be the expected return on this asset?

3 Portfolio manager evaluation

Consider an economy where CAPM holds. There are many risky assets and one riskfree asset with interest rate is $r_f = 0$. The market portfolio of risky assets has an expected return of 7.5% and a standard deviation of 15%. An investor (who has linear-quadratic preferences) considers allocating his/her entire wealth to a portfolio manager who runs an investment fund, and two different candidate funds are being considered.

1. The two candidates can deliver portfolios with different means and standard deviations. Explain why it is sufficient to know the mean and standard deviation of the returns on the candidate funds in order for the investor to rank them. In particular, explain what criterion the investor should use to evaluate and rank the candidate funds.

2. Candidate A can deliver returns with mean $E(R_{p1}) = 6\%$ and standard deviation $\sigma(R_{p1}) = 12\%$, while candidate B can deliver $E(R_{p1}) = 5\%$ and standard deviation $\sigma(R_{p1}) = 8\%$. Which fund should the investor choose? Explain your answer.

3. Explain why all owners with linear-quadratic preferences would choose the same portfolio manager, irrespectively of their risk aversion.

4   Consumption-based asset pricing

Consider a representative agent economy where all households consume the same amount c_t (and this amount c_t can vary over time). Households have preferences

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \right\}.$$

There exists many stocks with stochastic returns and a risk-free asset with return r_f. Define the return on the stock R_{it} as

$$R_{it} = \frac{P_{it} + D_{it}}{P_{i,t-1}},$$

where P_{it} is the price of the asset in period t and D_{it} is the dividend of the asset.

1. Explain in words why households will invest such that in equlibrium the following equation is satisfied:

$$P_{it} = E_t \left\{ \beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} \left(P_{i,t+1} + D_{i,t+1}\right) \right\}$$

2. Suppose one stock (Forest Harvest) has a stream of future dividends which is completely independent of aggregate consumption. Note that in this case the return on Forest Harvest, R_{it}, will be independent of c_t. What will be the expected return for Forest Harvest? Explain your answer

3. Suppose preferences are CRRA, $u\left(c\right) = c^{1-\gamma}/\left(1-\gamma\right)$, and that the consumption growth c_{t+1}/c_t and returns on stocks are all jointly log-normal. Moreover, suddenly news arrive that announces that future dividends for Forest Harvest will become highly correlated with aggregate consumption, while the expected value of the dividends will remain unchanged. What will happen to the price of the stock of Forest Harvest and to the expected return on Forest Harvest stocks?

4. Suppose households have CRRA risk aversion $\gamma = 5$. Moreover, their stochastic process for consumption is equal to the process for aggregate consumption in the U.S. and the process return on stocks are the same as for the U.S. economy, including an expected return on stocks of $E\left(R_{M,t}\right) = 7\%$. Explain why an equilibrium in this economy can be expected to deliver a return on bonds which is much larger than the average empirical return on bonds in the U.S. (which is close to 1%).