Solution Manual for econ4510 Exam
May 2021

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1 CAPM (45%)

Assume that investors have linear-quadratic preferences. There are many risky assets.

a. Assume first that there is no risk free asset. Explain, with the use of a diagram, the trade-off between risk and return for the investors. In particular, draw the investors' indifference curves and the opportunity set of possible portfolios.

   Answer: Draw convex indifference curves in the mean-standard deviation diagram. Indifference curves must be flat when crossing the y-axis (with $\sigma = 0$). Plot the location of individual stocks as dots and draw the efficiency frontier as a hyperbola (minimum standard deviation conditional on mean).

b. Consider a stock with expected return $\bar{r}_j$ and standard deviation of return σ_j. Explain why this stock must have more risk (higher σ_j) than the portfolio of the efficiency frontier which has the same expected return $\bar{r}_j$.

   Answer: The efficiency frontier is obtained by minimizing the variance conditional on mean. Therefore, by construction, the efficiency frontier must have a lower variance than any stock that yields the same expected return.

c. Suppose a risk free asset with return r_f is introduced. How will this change the opportunity set for the investors? Explain why all investors will purchase a combination of two portfolios, even though they may have different risk aversion. What must these portfolios be in equilibrium?

   Answer: By combining the risk free asset and the best possible portfolio at the efficiency frontier, the investors can do better than any point on the efficiency frontier (use a graphical argument). Illustrate graphically that this holds true regardless of the risk aversion. This result implies a two-fund separation: all investors combine the risk free asset with the best portfolio on the efficiency frontier. Since all investors purchase the same portfolio of risky assets, this portfolio must be the market portfolio.

d. Explain how the equilibrium in this economy implies the CAPM formula for asset-pricing.

   Answer: Use argument of calculating the standard deviation of a portfolio combining the market portfolio and an individual stock. Let the weight

on the market portfolio go to 1. Exploit that the Capital market Line must be a tangent to the efficiency frontier evaluated at the market portfolio. Exploiting a differential argument, this implies the CAPM formula:

$$E(r_j) - r_f = \beta_j (E(r_M) - r_f),$$

where

$$\beta = \frac{cov(r_j, r_M)}{var(r_M)}$$

e. Explain – in light of Roll's critique – why it is difficult to test the empirical validity of CAPM.

Answer: A rejection of CAPM would involve finding stocks that are not located at the Security Market Line. Testing CAPM requires knowledge of the return on the market portfolio in order to calculate SML. This portfolio is inherently difficult to observe because it involves more assets than just those traded at the stock exchange (the true market portfolio must include human capital, residential housing, etc.). If one were to use an incomplete portfolio as a stand-in for the market portfolio (for example the index for the stocks traded at the stock exchange), then one might find evidence of a rejection of CAPM even though CAPM might be true.

f. Assume that over the last five years, the average real return on the market portfolio has been 9% while the standard deviation of this return has been 10%. Moreover, the risk free rate has been 1% over this period. The stock Equinor has had a covariance of 0.009 with the market portfolio over the last five years. Calculate the market β for Equinor and the expected return on the Equinor stock, according to CAPM.

Answer:

$$\begin{aligned} \beta_{Eq} &= \frac{cov(r_{Eq}, r_M)}{var(r_M)} = \frac{0.009}{(0.1)^2} = 0.9 \\ E(r_{Eq}) &= r_f + \beta_{Eq}(E(r_M) - r_f) = 0.01 + 0.9 * (0.09 - 0.01) = 8.2\% \end{aligned}$$

2 Option pricing (40%)

A stock price is currently NOK 100. It is known that at over the next month the price will either increase by 6% or fall by 5%. The following month the stock price will have the same evolution in prices (up by 6% or down by 5%). The risk-free interest rate is 6% per annum with continuous compounding.

a. What is the value of a one-month European call option with a strike price of NOK 102?

Answer: The values at expiration are $c_u = 106 - 102 = 4$ and $c_d = \max\{0, 95 - 102\} = 0$. Moreover, define the risk-neutral probability as

$p = (\exp(0.06/12) - 0.95)/(1.06 - 0.95) \approx 0.5$. Using an absence of arbitrage argument the formula for the one-period option price can be derived as

$$c = \frac{pc_u + (1-p)\,c_d}{\exp(r)} = \frac{\frac{1}{2}\cdot 4 + \left(1-\frac{1}{2}\right)\cdot 0}{\exp(0.06/12)} \approx 1.99$$

b. Consider a two-month European call option with a strike price of NOK 102. What is the value of this option?

Answer: The values at expiration are $c_{u,u} = 100 * (1.06)^2 - 102 = 10.36$, $c_{ud} = \max\{0, 100 * 1.06 * 0.95 - 102\} = 0$, and $c_{dd} = 0$. This implies

$$\begin{aligned} c_u &= \frac{pc_{uu} + (1-p)c_{du}}{e^r} = \frac{\frac{1}{2} * 10.36}{\exp(0.06/12)} \approx 5.154 \\ c_d &= \frac{pc_{ud} + (1-p)c_{dd}}{e^r} = 0 \\ c &= \frac{pc_u + (1-p)\,c_d}{\exp(r)} = \frac{\frac{1}{2}\cdot 5.154}{\exp(0.06/12)} \approx 2.564 \end{aligned}$$

c. Explain how a put option can be constructed using a call option and the underlying stock.

Answer: Consider following set of four transactions:

		At expiration	
	Now	If $S_T \le K$	If $S_T > K$
Sell call option	c	0	$K - S_T$
Buy put option	$-p$	$K - S_T$	0
Buy share	$-S$	S_T	S_T
Borrow (risk free)	Ke^{-rT}	$-K$	$-K$
Total	$c - p - S + Ke^{-rT}$	0	0

Must have $c = p + S - Ke^{-rT}$, if not, riskless arbitrage.

d. What is the value of a one-month European put option with a strike price of NOK 102?

Answer: $p = c - S + Ke^{-rT} = 1.99 - 100 + 102 * \exp(0.06/12) \approx 4.50$.

3 Portfolio management (15%)

Over the last five years, the Norwegian Oil Fund has had an annual return on stocks of 10.68%. The corresponding number for the benchmark index (i.e., the benchmark portfolio the fund is asked to follow – which is the global index of stocks) is 10.53%. The *Relative Return* is therefore 10.68%-10.53%=0.15%. During the same period, the Jensen's Alpha[1] for the fund was 0.01%. These measures are all recorded before costs. The costs of the management of stocks was on average 0.05% during this period.

[1] Jensen's Alpha is defined as "Average return in excess of the risk-free rate minus beta-adjusted benchmark excess gross-of-fees return."

a. Explain why Jensen's Alpha is a better measure of performance than the Relative Return.

Answer: A positive relative return on average could be achieved by either having superior skill or by taking excessive systematic risk. The latter could be achieved by exploiting the tracking error to either choose a portfolio with a higher β than the benchmark or by choosing more stocks and less bonds than in the benchmark. The Jensen's Alpha corrects for the exposure to market risk and is therefore a better measure of skill and performance.

b. Define "active management" as any deviation from the benchmark portfolio. How would you judge the active management of the Oil Fund over the last five years?

Answer: The fact that Jensen's Alpha is approximately zero (before costs) suggests that the performance of active management of stocks was no better than just pure index investing (i.e., following the index passively). Active management is more expensive than passive management. Therefore, the performance of active management after costs has been slightly worse than just a passive investment strategy.

c. The tracking error for the Oil Fund is 0.44% and the Sharpe ratio for the fund is approximately equal to the Sharpe ratio for the benchmark (0.72 over the last five years). In light of this, please evaluate the statement that "The Norwegian Oil Fund is, for all practical purposes, an index fund."

Answer: A very small tracking error suggests that the difference between the actual portfolio and the benchmark portfolio is very small. The fact that the Sharpe ratio is the same for the benchmark and the actual confirms that the Norwegian Oil Fund is, for all practical purposes, an index fund.