Final Exam ECON4510 «Finance Theory» Spring -22

(10 points) 1. *Expected equity return, average cost of capital, and leverage*

You have a firm that uses only equity to finance its activities. Its expected returns are 10%. Assume the firm instead considers the following financing structures:

- 10% debt, 90% equity;
- 50% debt, 50% equity;
- 90% debt, 10% equity.

For simplicity, assume that the cost of debt is 5% and the same in all three cases

(a) What are the firm's expected returns on its equity for each of the financing structures the firm considers?

(b) What is the (weighted) average cost of capital in each of the three cases cases?

(15 points) 2. *Capital budgeting*

Your organization has two potential projects with the following projected cash flows:

"*Project A*":

	Year 0	Year 1	Year 2	Year 3	Year 4
Initial investment	-1,000,000				
Sales		120,000	700,000	900,000	800,000
Cost of goods sold		100,000	300,000	400,000	300,000

"*Project B*":

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Initial investment	-1,000,000					
Sales		800,000	800,000	800,000	800,000	
Cost of goods sold		400,000	400,000	400,000	400,000	
Salvage costs						-400,000

Salvage costs are the costs involved in salvaging goods or property. Salvage value would have been the amount that an asset is estimated to be worth at the end of its useful life.

The risk-free rate is 2%, and the expected market risk premium is 5%, and The beta of "Project A" is 1.8, and the beta of "Project B" is 2.0.

Your organization has only \$ 1,000,000 available for investment in this year's budget, so it has to decide which of the two projects to choose.

(a) What is the cost of capital for each of the potential projects?

(b) Which project should your organization choose?

(c) Which project is chosen based on the payback rule?

(10 points) 3. *Time value of money and the yield curve*

(a) You have three risk-free zero-coupon bonds. They all have a face value of NOK 1,000. The first bond is a one-year bond and has a price of NOK 950. The second is a two-year bond and has a current price of NOK 900. The third one is a three-year bond and has a current price of NOK 800. Compute the yield curve up to three years based on this information.

(b) You have a three-year bond with annual coupons paid at the end of the year, the next coupon one year from now. The coupon rate is 6%, and the face value is NOK 1,000. What is the price of the bond?

(10 points) 4. *Expected returns, and risk premia*

You have the following information about assets A, B, C and D:

		Price next year		
Asset	Current price	Good economy	Average economy	Bad economy
A	100	108	104	98
B	205	208	206	204
C	320	304	306	320
D	115	138	119	100

A, B and D are shares that are not expected to pay any dividends over the next 12 months. C is a share that is expected to pay a dividend of 4 per share if the economy is good or average. (The dividend has the same time horizon as the "price next year".)

The probabilities of having a good/average/bad economy over the next year are generally thought to be 25%, 50% and 25% respectively.

(a) What are the expected returns on each of the assets?

(b) If the annual interest rate is 1%, what are their risk premia?

(10 points) 5. *Dividends and repurchases*

You own 2% of a firm worth NOK 20,000,000.

(a) Suppose NOK 2,000,000 is paid out as dividend. How much do you get? How much are your shares worth after the distribution of the dividend?

(b) The firm repurchases 10% of its shares (it pays NOK 2,000,000 for them). You decide to sell 10% of your shares back to the firm. What is the market value of your wealth? How about when you sell all your shares – or when you do not sell any of them?

(10 points) 6. *Expected future returns*

The value of a financial asset (or a composite of financial assets, like an index) is the net present risk-adjusted value of all future cash flows

$$P_0 = \mathrm{Div}_0 + \frac{\mathrm{Div}_1}{1+r_E} + \frac{\mathrm{Div}_2}{(1+r_E)^2} + \frac{\mathrm{Div}_3}{(1+r_E)^3} + \frac{\mathrm{Div}_4}{(1+r_E)^4} + \cdots = \sum_{n=0}^{\infty} \frac{\mathrm{Div}_n}{(1+r_E)^n}$$

If we assume that dividends grow at a constant (and known) rate μ such that

$$\mathrm{Div}_1 = \mathrm{Div}_0(1+\mu),\ \mathrm{Div}_2 = \mathrm{Div}_0(1+\mu)^2,\ \text{and so on,}$$

show that

$$r_E = \frac{\mathrm{Div}_0}{P_0} + \mu$$

Remember that the sum of an infinite geometric series is

$$s = a + ag + ag^2 + ag^3 + ag^4 + \cdots = \sum_{k=0}^{\infty} ag^k = \frac{a}{1-g},\ \text{for } |g| < 1.$$

and for x_1 and x_2 small

$$(1+x_1)\cdot(1+x_2) \approx 1 + x_1 + x_2$$

(15 points) 7. *Returns and risk premiums.* Consider the asset prices and payoffs

$$\begin{array}{lll} \text{Asset 1}: & p_1 = 3/4, & \tilde{x}_1(1) = 1,\ \tilde{x}_1(2) = 1 \\ \text{Asset 2}: & p_2 = 1, & \tilde{x}_2(1) = 1,\ \tilde{x}_2(2) = 2. \end{array}$$

(a) What are the state prices? Is $(p, \tilde{x})$ arbitrage free?

(b) What are the returns on the two assets?

(c) If $\mathrm{prob}_1 = \mathrm{prob}_2 = 1/2$, what are the expected returns?

(d) What is the stochastic discount factor? Why does the second asset have a higher excess return?

(e) What are the risk-neutral probabilities? Why does the second asset have a higher excess return?

(20 points) 8. *The risk-free rate and intertemporal consumption choice*

Consider the two-period decision problem: choose consumptions (c_0, c_1) to maximize utility

$$U(c_0, c_1) = \frac{c_0^{1-\rho}}{1-\rho} + \beta \frac{c_1^{1-\rho}}{1-\rho}$$

subject to the budget constraint $c_0 + \frac{1}{R_f} c_1 \leq y_0$ (that is, there's no date-1 income).

(a) What is the Lagrangian function associated with this problem? What are the first-order conditions?

(b) Express consumptions as functions of income y_0 and the risk-free rate R_f.

(c) How does date-0 consumption c_0 depend on R_f?

(d) How does saving depend on R_f? On the interest rate?

(e) *For given* c_0 and c_1, show that the solution to the agent's optimization problem implies that R_f must be lower when the elasticity of intertemporal substitution is higher. Relate to the so-called "risk-free rate puzzle".