Final Exam ECON4510 «Finance Theory» Spring 2023

(10 points) 1. *Bond pricing*

(a) You have three risk-free zero-coupon bonds. They all have a face value of $ 1,000. The first bond is a one-year bond and has a price of $ 950. The second is a two-year bond and has a current price of $ 900. The third one is a three-year bond and has a current price of $ 800. Provide the points on the yield curve based on this information.

(b) You have a three-year bond with annual coupons paid at the end of the year, the next coupon one year from now. The coupon rate is 6%, and the face value is $ 1,000. What is the price of the bond?

(15 points) 2. *Options pricing, binomial model*

The current price of an Equinor stock is 302 kroner. Compute the price of a one-year call option with strike 300 kroner using a two-period binomial approximation.

During each of the next six months, the price of the stock may increase or decrease by 25 kroner. The annual interest rate is 3.6%. You may assume that the yield curve is flat.

(15 points) 3. *"Ground rent taxation"*

(a) Suppose a firm has total sales of 120 Million, cost of goods sold is 90 Million, and make new investments of 10 Million per year. What is the firm's net free cash flow?

(b) Suppose sales, cost, and investment all grow at an annual rate of 2 percent per year in perpetuity, and that the discount rate on the firm's free cash flows is 7 percent. What is the value of the firm?

(c) Suppose the government suddenly introduces a new tax, euphemistically called a "ground rent tax", on net free cash flow of 35 percent. What is the value of the firm after the announcement of the new tax?

(d) Further suppose the government defended the tax by referring it as "neutral". Neutral taxes are taxes which do not cause inefficiency by distorting the structure of incentives. Poll taxes and lump-sum taxes are neutral. Based on your findings in (c), how is the proposed "ground rent tax" neutral?

(15 points) 4. *The CAPM*

(a) Klippfisk shares have a beta of 1.5. The expected risk-free rate is 3%, and the expected returns on the market portfolio are 9%. In equilibrium, supposing that the CAPM holds, what should be the expected return on Klippfisk shares?

(b) As mentioned above, Klippfisk shares have a beta of 1.5. Tørrfisk shares have a beta of 1.2. You invest half of your money in Klippfisk and half of your money in Tørrfisk. What are your expected returns?

(c) The shares of Bacalao ASA have a beta of 0.6 and based on their current price the expected return is 8%. Are the shares overpriced, underpriced or priced correctly? If they are not priced correctly, how could you make money from this information?

(15 points) 5. *Absence of arbitrage*

Our starting point is the no-arbitrage condition in competitive markets

$$\mathbf{E}_t[m_{t+1} \cdot R_{t+1}] = 1$$

(a) Step by step, show that

$$\begin{aligned}\mathbf{E}R_{t+1} &= \frac{1}{\mathbf{E}_t[m_{t+1}]} - \frac{\text{cov}[m_{t+1}, R_{t+1}]}{\mathbf{E}_t[m_{t+1}]} \\ &= R^f_{t+1} - R^f_{t+1}\text{cov}[m_{t+1}, R_{t+1}]\end{aligned}$$

where we have used that $R^f_{t+1} = \frac{1}{\mathbf{E}_t[m_{t+1}]}$.
Briefly explain what the two right-hand-side terms are.

(b) In currency markets,

$$R_{t+1} = \frac{S_{t+1}}{S_t}\frac{1+i^*}{1+i}$$

Step by step, show that

$$\mathbf{E}_t\left[\frac{S_{t+1}}{S_t}\right] = \frac{1}{\mathbf{E}_t\left[m_{t+1}\right]}\left(\frac{1+i}{1+i^*} - \text{cov}\left(m_{t+1}, \frac{S_{t+1}}{S_t}\right)\right)$$

and

$$\begin{aligned}\mathbf{E}_t S_{t+1} &= \frac{1}{\mathbf{E}_t\left[m_{t+1}\right]}\left(S_t\frac{1+i}{1+i^*} - \text{cov}\left(m_{t+1}, S_{t+1}\right)\right) \\ &= R^f_{t+1} \cdot \left(F_{t+1} - \text{cov}\left(m_{t+1}, S_{t+1}\right)\right)\end{aligned}$$

where the forward rate, $F_{t+1} \equiv S_t\frac{1+i}{1+i^*}$

(c) On January 1st 2023, the NOK/EUR exchange rate was 10.51, and 3M interbank rates were 3.27 and 2.34 in NOK and EUR, respectively. Interest rates are in percent in annual terms. On April 1st, 2023 the NOK/EUR exchange rate was 11.39. Briefly discuss the source of the exchange rate change in light of the equations in (b).

(15 points) 6. *State prices and related objects.* Consider an economy with three states. State prices and probabilities are

State z	State Price $Q(z)$	Probability $p(z)$	Dividend $d(z)$
1	1/2	1/3	1
2	1/3	1/3	2
3	1/4	1/3	3

(a) What is the pricing kernel in each state?

(b) What is the price of a one-period bond? What is its return?

(c) What are the risk-neutral probabilities? Why are they different from the true probabilities?

(d) Suppose equity is a claim to the dividend in the last column. What is its price? What is the return on equity in each state?

(e) What is the expected return on equity? The risk premium?

(15 points) 7. *Absolute equilibrium pricing*

(a) Starting from the general no-arbitrage condition

$$\mathbf{E}_t\left(m_{t+1}R_{t+1}\right) = 1$$

show the steps to arrive at the HJ bounds

$$\frac{\sigma(m_{t+1})}{\mathbf{E}_t m_{t+1}} \geq \frac{\mathbf{E}_t(R_{t+1} - R^f_{t+1})}{\sigma(R_{t+1} - R^f_{t+1})}$$

(b) Using monthly returns from Shiller's database, we computed the Sharpe-ratio $((R_{t+1} - R^f_{t+1})/\sigma(R_{t+1} - R^f_{t+1}))$ to be 0.225.
In class we referred to the Sharpe ratio as "the market price of risk". Please briefly explain what we meant by that. Please also make a reference to the capital market line in the CAPM.

(c) Assuming standard power utility

$$U_t = \max \mathbf{E}_t \sum_t \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

subject to a parsimonious budget constraint, show that

$$m_{t+1} = \beta \Delta C_{t+1}^{-\gamma}$$

(d) Using monthly real U.S. consumption data from the BEA obtained through FRED, for $\beta = .999$ we found that we needed $\gamma = 38$ to satisfy the HJ bounds. What risk-free rate would this γ imply?

(e) Briefly explain the asset pricing puzzles and the failure of standard macroeconomic models to simultaneously account for the dynamics of consumption (and other macroeconomic variables), the risk-free rate, and the risk premium / the market price of risk.

(f) Briefly mention a couple of approaches to build models that can account for the dynamics of consumption, the risk-free rate, and the risk premium / the market price of risk.