Final Exam ECON4510 «Finance Theory» Spring 2023

Solution: Answers will follow. For instructional purposes, these may be more elaborate than what was expected from the students writing the final exam.

(10 points) 1. *Bond pricing*

(a) You have three risk-free zero-coupon bonds. They all have a face value of \$ 1,000. The first bond is a one-year bond and has a price of \$ 950. The second is a two-year bond and has a current price of \$ 900. The third one is a three-year bond and has a current price of \$ 800. Provide the points on the yield curve based on this information.

(b) You have a three-year bond with annual coupons paid at the end of the year, the next coupon one year from now. The coupon rate is 6%, and the face value is \$ 1,000. What is the price of the bond?

Solution:

(a) The one-year annual yield is $\frac{1000}{950} - 1 = 5.26\%$.

The two-year annual yield is $\frac{1000}{900}^{\frac{1}{2}} = 5.41\%$.

The three-year annual yield is $\frac{1000}{800}^{\frac{1}{3}} = 7.72\%$

(b) The annual coupon is $0.06 \times 1,000 = \$60$. The price of the bond is the present value of its future cash flows.

$$P_{\text{bond}} = \frac{60}{1.0526} + \frac{60}{1.0541^2} + \frac{1060}{1.0772^3} = 959.$$

(15 points) 2. *Options pricing, binomial model*

The current price of an Equinor stock is 302 kroner. Compute the price of a one-year call option with strike 300 kroner using a two-period binomial approximation.

During each of the next six months, the price of the stock may increase or decrease by 25 kroner. The annual interest rate is 3.6%. You may assume that the yield curve is flat.

Solution:

1. Solve first for the case if the stock price increased by 25 kroner during the first six-months period.

   We then have the two equations:

$$(1+r)\cdot B + \Delta \cdot 352 = 52$$
$$(1+r)\cdot B + \Delta \cdot 302 = 2$$

   First solving for Δ

$$\Delta = 1$$

   and then for B

$$(1+r)\cdot B + 352 = 52$$

   so

$$B = \frac{-300}{1.018} = -294.8$$

   The value of the option after six months, if the stock price is 327, would have been:

$$C^H = B + 327 \cdot \Delta = -294.8 + 327 \cdot 1 = 32.2$$

2. Solve then for the case if the stock price decreased by 25 kroner during the first six-months period. We then have the two equations:

$$(1+r)\cdot B + \Delta \cdot 302 = 2$$
$$(1+r)\cdot B + \Delta \cdot 252 = 0$$

First solving for Δ

$$\Delta = \frac{2}{50} = 0.04$$

and then for B

$$(1+r)\cdot B + 302\cdot 0.04 = 2$$

so

$$B = \frac{2-12.08}{1.018} = \frac{-10.08}{1.018} = -9.9$$

The value of the option after six months, if the stock price is 277, would have been:

$$C^L = B + 277\cdot\Delta = -9.9 + 277\cdot 0.04 = 1.2$$

3. And finally solving for the value of the option at initiation

   We then have the two equations:

$$(1+r)\cdot B + \Delta\cdot 327 = C^H$$
$$(1+r)\cdot B + \Delta\cdot 277 = C^L$$

First solving for Δ

$$\Delta = \frac{C^H - C^L}{327-277} = \frac{32.2-1.2}{50} = 0.62$$

and then for B

$$(1+r)\cdot B + 327\cdot 0.62 = 32.2$$

so

$$B = \frac{32.2-202.7}{1.018} = \frac{-170.5}{1.018} = -167.5$$

The value of the option at initiation is

$$C = B + 302\cdot\Delta = -167.5 + 302\cdot 0.62 = 19.7$$

(15 points) 3. *"Ground rent taxation"*

(a) Suppose a firm has total sales of 120 Million, cost of goods sold is 90 Million, and makes new investments of 10 Million per year. What is the firm's net free cash flow?

(b) Suppose sales, cost, and investment all grow at an annual rate of 2 percent per year in perpetuity, and that the discount rate on the firm's free cash flows is 7 percent. What is the value of the firm?

(c) Suppose the government suddenly introduces a new tax, euphemistically called a "ground rent tax", on net free cash flows of 35 percent. What is the value of the firm after the announcement of the new tax?

(d) Further suppose the government defended the tax by referring to it as "neutral". Neutral taxes are taxes that do not cause inefficiency by distorting the structure of incentives. Poll taxes and lump-sum taxes are neutral. Based on your findings in (c), how is the proposed "ground rent tax" neutral?

Solution:

(a)

$$\text{Net free cash flow} = 120 - 90 - 10 = 20$$

(b)

$$V = \frac{20}{0.07 - 0.02} = 400$$

(c) The free cash flow is now $20 \cdot 0.65 = 13$

The new value of the firm's assets is

$$\hat{V} = \frac{20 \cdot 0.65}{0.07 - 0.02} = 0.65 \cdot \frac{20}{0.07 - 0.02} = 0.65 \cdot V = 0.65 \cdot 400 = 260$$

In general for a tax rate τ on net free cash flows

$$\hat{V} = (1 - \tau) \cdot V$$

(d) In a hypothetical static world, the proposed "ground rent tax" is "neutral" for the exact same reason as poll taxes and lump-sum taxes are "neutral"; as shown in (c) the proposed tax is equivalent to a lump-sum confiscation of 35% of the value of the firm.

(15 points) 4. *The CAPM*

(a) Klippfisk shares have a beta of 1.5. The expected risk-free rate is 3%, and the expected returns on the market portfolio are 9%. In equilibrium, supposing that the CAPM holds, what should be the expected return on Klippfisk shares?

(b) As mentioned above, Klippfisk shares have a beta of 1.5. Tørrfisk shares have a beta of 1.2. You invest half of your money in Klippfisk and half of your money in Tørrfisk. What are your expected returns?

(c) The shares of Bacalao ASA have a beta of 0.6 and based on their current price the expected return is 8%. Are the shares overpriced, underpriced or priced correctly? If they are not priced correctly, how could you make money from this information?

Solution:

(a) We know that expected / required rate of return on an individual project / stock is

$$\mathbf{E}[R_i] = R^f + \beta_i \left(\mathbf{E}[R^m] - R^f\right)$$

Here

$$\begin{aligned}\mathbf{E}[R_{\text{Klippfisk}}] &= R^f + \beta_{\text{Klippfisk}} \left(\mathbf{E}[R^m] - R^f\right) \\ &= 0.03 + 1.5 \cdot 0.06 = 0.12 = 12\%\end{aligned}$$

(b)

$$\begin{aligned}\mathbf{E}[R_{\text{Tørrfisk}}] &= R^f + \beta_{\text{Tørrfisk}} \left(\mathbf{E}[R^m] - R^f\right) \\ &= 0.03 + 1.2 \cdot 0.06 = 0.102 = 10.2\%\end{aligned}$$

$$0.5 \cdot \mathbf{E}[R_{\text{Klippfisk}}] + 0.5 \cdot \mathbf{E}[R_{\text{Tørrfisk}}] = 0.112 = 11.2\%$$

(c)

$$\begin{aligned}\mathbf{E}[R_{\text{Bacalao}}] &= R^f + \beta_{\text{Bacalao}} \left(\mathbf{E}[R^m] - R^f\right) \\ &= 0.03 + 0.6 \cdot 0.06 = 0.066 = 6.6\%\end{aligned}$$

The stock is underpriced, and we could, in risk-adjusted expected terms, make money buying it.

(15 points) 5. *Absence of arbitrage*

Our starting point is the no-arbitrage condition in competitive markets

$$\mathbf{E}_t[m_{t+1} \cdot R_{t+1}] = 1$$

(a) Step by step, show that

$$\begin{aligned} \mathbf{E}R_{t+1} &= \frac{1}{\mathbf{E}_t[m_{t+1}]} - \frac{\text{cov}[m_{t+1}, R_{t+1}]}{\mathbf{E}_t[m_{t+1}]} \\ &= R^f_{t+1} - R^f_{t+1}\text{cov}[m_{t+1}, R_{t+1}] \end{aligned}$$

where we have used that $R^f_{t+1} = \frac{1}{\mathbf{E}_t[m_{t+1}]}$.
Which of these terms is the risk premium?

(b) In currency markets,

$$R_{t+1} = \frac{S_{t+1}}{S_t}\frac{1+i^*}{1+i}$$

Step by step, show that

$$\mathbf{E}_t\left[\frac{S_{t+1}}{S_t}\right] = \frac{1}{\mathbf{E}_t\left[m_{t+1}\right]}\left(\frac{1+i}{1+i^*} - \text{cov}\left(m_{t+1}, \frac{S_{t+1}}{S_t}\right)\right)$$

and

$$\begin{aligned} \mathbf{E}_t S_{t+1} &= \frac{1}{\mathbf{E}_t\left[m_{t+1}\right]}\left(S_t\frac{1+i}{1+i^*} - \text{cov}\left(m_{t+1}, S_{t+1}\right)\right) \\ &= R^f_{t+1} \cdot \left(F_{t+1} - \text{cov}\left(m_{t+1}, S_{t+1}\right)\right) \end{aligned}$$

where the forward rate, $F_{t+1} \equiv S_t\frac{1+i}{1+i^*}$

(c) What assumption is (almost always implicitly) made in the so-called "uncovered interest rate parity hypothesis"?

(d) On January 1st 2023, the NOK/EUR exchange rate was 10.51, and 3M interbank rates were 3.27 and 2.34 in NOK and EUR, respectively. Interest rates are in percent in annual terms. On April 1st, 2023 the NOK/EUR exchange rate was 11.39. Briefly discuss the source of the exchange rate change in light of the equations in (b).

Solution:

(a)

$$\mathbf{E}_t[m_{t+1} \cdot R_{t+1}] = 1$$

$$\mathbf{E}_t[m_{t+1}] \cdot \mathbf{E}_t[R_{t+1}] + \text{cov}\left[m_{t+1}, R_{t+1}\right] = 1$$

$$\mathbf{E}_t[m_{t+1}] \cdot \mathbf{E}_t[R_{t+1}] = 1 - \operatorname{cov}[m_{t+1}, R_{t+1}]$$

$$\mathbf{E}_t[R_{t+1}] = \frac{1}{\mathbf{E}_t[m_{t+1}]} - \frac{\operatorname{cov}[m_{t+1}, R_{t+1}]}{\mathbf{E}_t[m_{t+1}]}$$

Since $R^f_{t+1} = (\mathbf{E}_t[m_{t+1}])^{-1}$

$$\mathbf{E}_t[R_{t+1}] = R^f_{t+1} - R^f_{t+1}\operatorname{cov}[m_{t+1}, R_{t+1}]$$

(b)

$$\mathbf{E}_t[m_{t+1}R_{t+1}] = 1$$

$$\mathbf{E}_t\left[m_{t+1}\frac{S_{t+1}}{S_t}\frac{1+i^*}{1+i}\right] = 1$$

$$\frac{1}{S_t}\frac{1+i^*}{1+i}\mathbf{E}_t[m_{t+1}S_{t+1}] = 1$$

$$\left(\frac{1}{S_t}\frac{1+r^*}{1+r}\right)(\mathbf{E}_t[m_{t+1}]\,\mathbf{E}_t[S_{t+1}] + \operatorname{cov}(m_{t+1}, S_{t+1})) = 1$$

$$\mathbf{E}_t[m_{t+1}]\,\mathbf{E}_t[S_{t+1}] + \operatorname{cov}(m_{t+1}, S_{t+1}) = S_t\frac{1+i}{1+i^*}$$

$$\mathbf{E}_t[m_{t+1}]\,\mathbf{E}_t[S_{t+1}] = S_t\frac{1+i}{1+i^*} - \operatorname{cov}(m_{t+1}, S_{t+1})$$

Expected change in future spot:

$$\mathbf{E}_t\left[\frac{S_{t+1}}{S_t}\right] = \frac{1}{\mathbf{E}_t[m_{t+1}]}\left(\frac{1+i}{1+i^*} - \operatorname{cov}\left(m_{t+1}, \frac{S_{t+1}}{S_t}\right)\right)$$

Expected future spot:

$$\begin{aligned}\mathbf{E}_t S_{t+1} &= \frac{1}{\mathbf{E}_t[m_{t+1}]}\left(S_t\frac{1+r}{1+r^*} - \operatorname{cov}(m_{t+1}, S_{t+1})\right)\\ &= \frac{1}{\mathbf{E}_t[m_{t+1}]}(F_{t+1} - \operatorname{cov}(m_{t+1}, S_{t+1}))\end{aligned}$$

(c) An implicit assumption of the so-called “interest rate parity” or “uncovered interest rate parity” is that risk-premia are small and approximately constant.

(d) In this particular case, S is EURs in terms of NOKs, i^* is the Euro interest rate, and i is the NOK interest rate.

Sn quarterly terms,

$$\frac{1+i}{1+i^*} = \frac{1+\frac{0.0327}{4}}{1+\frac{0.0234}{4}} = \frac{1.008175}{1.00585} = 1.002311,$$

ie. a quartertly difference of 0.23 percentage points.

In contrast, the realized change in the spot rate during the quarter was

$$\frac{11.39}{10.51} = 1.0837$$

ie. a quarterly change of 8.37 percent.

The differences in interest rates are not remotely close to being able to quantitatively account for changes in spot exchange rates.

(15 points) 6. *State prices and related objects.* Consider an economy with three states. State prices and probabilities are

State z	State Price $Q(z)$	Probability $p(z)$	Dividend $d(z)$
1	1/2	1/3	1
2	1/3	1/3	2
3	1/4	1/3	3

(a) What is the pricing kernel in each state?

(b) What is the price of a one-period bond? What is its return?

(c) What are the risk-neutral probabilities? Why are they different from the true probabilities?

(d) Suppose equity is a claim to the dividend in the last column. What is its price? What is the return on equity in each state?

(e) What is the expected return on equity? The risk premium?

Solution:

State z	State Price $Q(z)$	Probs $p(z)$	Dividend $d(z)$	Pr.kernel $m(z)$	R-n probs $p^*(z)$	Return $r^e(z)$
1	1/2	1/3	1	3/2	0.4615	0.5217
2	1/3	1/3	2	1	0.3077	1.0435
3	1/4	1/3	3	3/4	0.2308	1.5652

(a) See the table.

(b) $q^1 = 1.0833, r^1 = 0.9231$.

(c) See table. They are a combination of the true probabilities p and the pricing kernel m, scaled to sum to one.

(d) The price is $q^e = 1.9167$. The returns are in the table.

(e) The expected return is $\mathbf{E}\left(r^e\right) = 1.0435$ and the risk premium is $\mathbf{E}\left(r^e - r^1\right) = 0.1204$. Why positive? You'll note that the asset's dividends are high when the pricing kernel is low, which makes the price low. That raises returns.

(15 points) 7. *Absolute equilibrium pricing*

(a) Starting from the general no-arbitrage condition

$$\mathbf{E}_t\left(m_{t+1} R_{t+1}\right) = 1$$

show the steps to arrive at the HJ bounds

$$\frac{\sigma(m_{t+1})}{\mathbf{E}_t m_{t+1}} \geq \frac{\mathbf{E}_t(R_{t+1} - R^f_{t+1})}{\sigma(R_{t+1} - R^f_{t+1})}$$

(b) Using monthly returns from Shiller's database, we computed the Sharpe-ratio $((R_{t+1} - R^f_{t+1})/\sigma(R_{t+1} - R^f_{t+1}))$ to be 0.225.
In class we referred to the Sharpe ratio as "the market price of risk". Please briefly explain what we meant by that. Please also make a reference to the capital market line in the CAPM.

(c) Assuming standard power utility

$$U_t = \max \mathbf{E}_t \sum_t \beta^t \frac{C_t^{1-\gamma} - 1}{1 - \gamma}$$

subject to a parsimonious budget constraint, show that

$$m_{t+1} = \beta \Delta C_{t+1}^{-\gamma}$$

(d) Using monthly real U.S. consumption data from the BEA obtained through FRED, for $\beta = .999$ we found that we needed $\gamma = 38$ to satisfy the HJ bounds. What risk-free rate would this γ imply?

(e) Briefly explain the asset pricing puzzles and why standard macroeconomic models fail to simultaneously account for the dynamics of consumption (and other macroeconomic variables), the risk-free rate, and the risk premium / the market price of risk.

(f) Briefly mention a couple of approaches to build models that can simultaneously account for the dynamics of consumption, the risk-free rate, and the risk premium / the market price of risk.

Solution:

(a) Starting point:

$$\mathbf{E}_t\left(m_{t+1}R_{t+1}\right) = 1$$

Using $R^f_{t+1} = 1/\mathbf{E}_t[m_{t+1}]$ we obtain

$$\mathbf{E}_t[m_{t+1}(R_{t+1} - R^f_{t+1})] = 0$$

m-discounted expected *excess* return for all assets is zero

Since $\mathbf{E}_t[m_{t+1}(R_{t+1} - R^f_{t+1})] = 0$,

$$\mathrm{cov}[m_{t+1}, R_{t+1} - R^f_{t+1}] = -\mathbf{E}_t[m_{t+1}]\mathbf{E}_t[R_{t+1} - R^f_{t+1}]$$

That is, risk premium or expected excess return

$$\begin{aligned}\mathbf{E}_t[R_{t+1} - R^f_{t+1}] &= -\frac{\mathrm{cov}[m_{t+1}, R_{t+1} - R^f_{t+1}]}{\mathbf{E}_t[m_{t+1}]} \\ &= -\frac{\mathrm{cov}[m_{t+1}, R_{t+1}]}{\mathbf{E}_t[m_{t+1}]} \\ &= -\frac{\rho[m_{t+1}, R_{t+1}]\sigma(R_{t+1})\sigma(m_{t+1})}{\mathbf{E}_t[m_{t+1}]}\end{aligned}$$

Rewritten in terms of Sharpe Ratio

$$\frac{\mathbf{E}_t[R_{t+1} - R^f_{t+1}]}{\sigma(R_{t+1})} = -\frac{\sigma(m_{t+1})}{\mathbf{E}_t[m_{t+1}]}\rho[m_{t+1}, R_{t+1}]$$

Since $\rho \in [-1, 1]$, we have

$$\frac{\sigma(m_{t+1})}{\mathbf{E}_t[m_{t+1}]} \geq \sup\left\|\frac{\mathbf{E}_t[R_{t+1} - R^f_{t+1}]}{\sigma(R_{t+1})}\right\|$$

(b) The "Sharpe ratio" measures the excess return per unit of deviation, and is a standard way of reporting the performance of an investment strategy.

It measured the expected return investors in competitive markets on the margin require to hold one more unit of risk, measured as standard deviation of returns.

The slope of the capital market line in the CAPM is the market's sharpe ratio; excess returns per unit of standard deviation.

(c) Given a generic intertemporal budget constraint, we will have the following first-order conditions

$$u'(c_t) = \lambda$$
$$\beta \mathbf{E}_t \left[u'(c_{t+1}) \cdot R_{t+1} \right] = \lambda$$

where λ is the shadow value of the intertemporal budget constraint.

Combining we get

$$\beta \mathbf{E}_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \cdot R_{t+1} \right] = 1$$

When utility is CRRA with risk-aversion coefficient γ

$$u'(c) = c^{-\gamma}$$

then

$$\beta \mathbf{E}_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \cdot R_{t+1} \right] = \beta \mathbf{E}_t \left[(\Delta C_{t+1})^{-\gamma} \cdot R_{t+1} \right] = 1$$

Defining $m_{t+1} = \beta \left(\Delta C_{t+1}\right)^{-\gamma}$, we have the familiar

$$\mathbf{E}_t \left[m_{t+1} \cdot R_{t+1} \right] = 1$$

(d) One piece of information was, unfortunately, missing from this question. The gross growth rate of consumption is about 1.015, ie. about 1.5 percent per year.

Then

$$R^f = \frac{1}{\mathbf{E}_t m_{t+1}} = \frac{1}{0.5673} = 1.7626$$

ie. the implied risk-free rate is 76.26 percent per year (!)

(e) "The equity premium puzzle" and "the risk-free rate puzzle" are twin puzzles.

The HJ bounds identified mechanism causing the equity premium puzzle — the inability of standard macro models to account for the risk premia observed in equity markets — as low volatility of the stochastic discount factor implied by macro models relative to its conditional mean.

The HJ bounds would be satisfied if the risk-aversion coefficient is sufficiently high. However, as we say from the previous question, the risk-free rate would then be too high. This is the “risk-free rate puzzle”.

(f) The two approaches we covered in class were:

1. Campbell-Cochrane external habits model
2. Epstein-Zin recursive utility and long-run risk