YOU MAY ANSWER IN ENGLISH OR NORWEGIAN.

Some advice: Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve. It is better to try to do something on each question than to get bogged down with one question. If you find you are spending too much time on one question, stop working on it and plan to get back to it if you have time at the end. Make sure you state any assumptions you make.

1. **Asset pricing [60%]**: Consider an economy where all households have linear-quadratic preferences $U_i(c) = \xi_i c - \phi_i c^2$ (but not necessarily the same preference coefficients ξ_i and ϕ_i). Each household i has some wealth to invest and has no other income. Moreover, assume that the households do not face any liquidity constraint (i.e., no binding borrowing constraint).

   (a) Suppose the household is offered two assets – a safe asset with return r_f and a risky asset with return $\tilde{r}$. Explain why the household will have a positive amount of the risky asset if and only if $E\{\tilde{r}\} > r_f$.
   SOLUTION: *Define the value function as* $W(a) \equiv E\{U[Y_0(1+r_f) + a(\tilde{r} - r_f)]\}$. *Consider second derivative:* $W''(a) = E\{U''[Y_0(1+r_f) + a(\tilde{r} - r_f)](\tilde{r} - r_f)^2\}$. *The function* $W(a)$ *will be concave since* U *is concave. Consider now the first derivative when* $a = 0$*:* $W'(0) = E\{U'([Y_0(1+r_f)](\tilde{r}-r_f)\} = U'[Y_0(1+r_f)]E(\tilde{r}-r_f)$. *Thus, if* $E(\tilde{r}) > r_f$*, then* $W'(0) > 0$*, which means that E(U) = W will be increased by increasing* a *from* $a = 0$*. The optimal* a *is thus strictly positive.*

   (b) Suppose the households can choose a portfolio comprising many risky assets (which are imperfectly correlated). Show that the household cares only about the expected return μ and the standard deviation σ of the portfolio.
   SOLUTION *Let the return on a portfolio of stocks and bonds be given by* $\tilde{W} = \sum_j \omega_j (1 + \tilde{r}_j)$*, where* ω_j *is weight on asset* j*. With this U function:*

$$\begin{aligned} E[U^i(\tilde{W})] &= \xi_i E(\tilde{W}) - \phi_i E(\tilde{W}^2) \\ &= -\phi_i\{E(\tilde{W}^2) - [E(\tilde{W})]^2\} - \phi_i[E(\tilde{W})]^2 + \xi_i E(\tilde{W}) \\ &= -\phi_i \mathrm{var}(\tilde{W}) - \phi_i[E(\tilde{W})]^2 + \xi_i E(\tilde{W}), \end{aligned}$$

   which is a function only of mean and variance of $\tilde{W}$*.*

   (c) Assume that one of the available assets is a risk free asset with return r_f.

      i. Illustrate the set of optimal portfolio choices for the households.
      SOLUTION: *Draw efficiency frontier in the mean-standard deviation diagram – linear capital market line*

ii. Show that all households will choose a combination of only two portfolios.
SOLUTION: *The capital market line is the set of efficient portfolios. Any point on the capital market line is comprised of two portfolios – the riskless bond and the market portfolio of stocks. It's useful to illustrate this with the aid of a diagram with indifference curves for two investors with different risk aversion.*

(d) Let $\tilde{r}_M$ denote the return on the market portfolio and let the covariance between the return on asset j and $\tilde{r}_M$ be given by $cov(\tilde{r}_M, \tilde{r}_j)$. Derive an expression for the expected return $E\{\tilde{r}_j\}$ in equilibrium. Provide an interpretation of the results.
SOLUTION:

$$E(\tilde{r}_j) - r_f = \beta_j(E(\tilde{r}_M) - r_f) = \frac{\sigma_{jM}}{\sigma_M^2}(E(\tilde{r}_M) - r_f),$$

Interpretation: "the expected excess rate of return on asset j equals its beta times the expected excess rate of return on the market portfolio. The higher the covariance of the asset with the market portfolio, the higher the beta. The investor gets compensated for taking systematic risk, i.e., risk correlated with the return on the market portfolio."

Derivation: Consider an equilibrium, everyone holds combination of risk free asset and market portfolio. Will derive relation between μ_j, σ_j (of any asset, numbered j) and the economy-wide variables r_f, μ_M, σ_M. As a thought experiment, make a portfolio with a fraction a in asset j and a fraction $1-a$ in the market portfolio (possible, even though M already contains j). For this portfolio p we have:

$$\mu_p = a\mu_j + (1-a)\mu_M, \quad \frac{\partial \mu_p}{\partial a} = \mu_j - \mu_M,$$

$$\sigma_p = \sqrt{a^2\sigma_j^2 + (1-a)^2\sigma_M^2 + 2a(1-a)\sigma_{jM}},$$

$$\frac{\partial \sigma_p}{\partial a} = \frac{a\sigma_j^2 - (1-a)\sigma_M^2 + (1-2a)\sigma_{jM}}{\sqrt{a^2\sigma_j^2 + (1-a)^2\sigma_M^2 + 2a(1-a)\sigma_{jM}}},$$

$$\left.\frac{\partial \sigma_p}{\partial a}\right|_{a=0} = \frac{\sigma_{jM} - \sigma_M^2}{\sigma_M}.$$

Small hyperbola goes through M (i.e., (σ_M, μ_M)). At M it has same tangent as large hyperbola: If not, it would have to cross over large hyperbola. But that cannot happen, since large hyperbola is frontier, and j was already available when large hyperbola was formed as frontier. The tangent is the capital market line. Next, use the equality of these slopes: Use the formula:

$$\frac{d\mu}{d\sigma}\frac{\partial\sigma}{\partial a} = \frac{\partial\mu}{\partial a} \iff \frac{d\mu}{d\sigma} = \frac{\frac{\partial\mu}{\partial a}}{\frac{\partial\sigma}{\partial a}}.$$

Use partial derivatives just found, evaluate at $a = 0$:

$$\left.\frac{\partial \sigma}{\partial a}\right|_{a=0} = \frac{\sigma_{jM} - \sigma_M^2}{\sigma_M}.$$

Plug in and find:

$$\left.\frac{d\mu}{d\sigma}\right|_{a=0} = \frac{\mu_j - \mu_M}{(\sigma_{jM} - \sigma_M^2)/\sigma_M}.$$

This slope of small hyperbola must equal slope of CML:

$$\frac{\mu_j - \mu_M}{(\sigma_{jM} - \sigma_M^2)/\sigma_M} = \frac{\mu_M - r_f}{\sigma_M} \iff \mu_j = r_f + (\mu_M - r_f)\frac{\sigma_{jM}}{\sigma_M^2}.$$

Known as the CAPM equation or the Security Market Line. Define $\beta_j \equiv \frac{\sigma_{jM}}{\sigma_M^2}$. Then rewrite as equation above.

(e) Suppose that households derive consumption from the return on the portfolio plus some labor income. Namely, each household works in one of the firms whose stocks are traded. Moreover, assume that the household's wage is positively correlated with the return on the stock of the firm they work for.

i. Will the result in 1.c.ii (that all households invest in two portfolios) survive? Substantiate your answer.
SOLUTION: *The two-fund separation result will break down. The reason is that each investor wants to short the stocks of the firm they work for in order to hedge their "background risk", i.e., their labor income.*

ii. What is the optimal portfolio now?
SOLUTION: *Idiosyncratic risk does not get priced. Therefore, it is optimal for the individual to short the asset so as to completely hedge the labor income risk. Once this is achieved, the individual will use the wealth (including the revenue from the shorted stock) to hold the market portfolio and the risk free asset.*

(f) Assume that the return on the market portfolio has $E\{\tilde{r}_M\} = 6\%$ and $std(\tilde{r}_M) = 10\%$ and that the risk free rate is $r_f = 1\%$. Consider now the return on a particular stock, say Yara. Assume that covariance between the return on the Yara stock, $\tilde{r}_Y$, and $\tilde{r}_M$ is given by $cov(\tilde{r}_Y, \tilde{r}_M) = 0.012$. Calculate the expected return on the Yara stock.
SOLUTION

$$\begin{aligned} E(\tilde{r}_j) &= r_f + \frac{\sigma_{jM}}{\sigma_M^2}(E(\tilde{r}_M) - r_f) \\ &= 0.01 + \frac{0.012}{(0.1)^2}(0.06 - 0.01) = 7\% \end{aligned}$$

(g) Suppose we learn that Yara has undertaken an investment (in a hotel in China, say) and that this investment will add new risks to the return $\tilde{r}_Y$. Namely, the new return on the Yara stock, $\tilde{R}_Y$,

$$\tilde{R}_Y = \tilde{r}_Y + \tilde{x},$$

where $\tilde{x}$ reflects the added risk associated with the hotel investment, so that the volatility of the return increases;. Moreover, suppose $corr\,(\tilde{x}, \tilde{r}_M) < 0$. How will the news influence the price of the Yara stock? Substantiate your answer.

SOLUTION *The investment will lower the covariance σ_{jM} between Yara and the market portfolio. This implies a lower β_j. which implies a lower expected return on the stock. This will in general imply an immediate increase in the Yara stock.*

(h) Suppose there is no safe asset. How does the optimal portfolios change? Is it possible that each household holds a different portfolio and that no individual household holds the market portfolio?

SOLUTION *The zero-covariance CAPM applies. The efficiency frontier is now the upper part of the hyperbola [draw a diagram]. That means that all households will hold portfolios on the efficiency frontier. However, households can be located at different points, depending on their risk aversion. Each point represents a different portfolio so households will in general hold portfolios different from the market portfolio (note that the market portfolio is just one point on the efficiency frontier).*

2. **Portfolio manager evaluation and empirical testing [25%]**:

(a) Suppose the household asks a portfolio manager to invest the funds. The household is interested in evaluating the portfolio manager, i.e., to determine if the manager is doing a good or a bad job.

i. Explain why the household needs an asset pricing theory in order to evaluate a portfolio manger

SOLUTION *In order to evaluate whether the portfolio manager has done a good or a bad job, it is necessary to have a comprehensive asset pricing theory. With such a theory in hand it is possible to judge whether the manager has achieved.*

ii. Suppose the household believes that CAPM is true. How should the portfolio manager be evaluated?

SOLUTION *The answer depends on whether the household has access to only one portfolio manager (in which case the Sharpe ratio the manager delivers is the right object) or whether there are many potential portfolio managers. In the latter case the manager should be evaluated on whether he/she can "beat the* *market", i.e., if Jensen's α is positive in a regression of the following type:*

$$\tilde{r}_{p,t} - r_f = \alpha_p + \beta_p(\tilde{r}_{M,t} - r_f) + \varepsilon_t$$

iii. The portfolio manager claims that he/she can "beat the market" since the expected return on his/her portfolio is higher than $E\{\tilde{r}_M\}$. Explain why the portfolio manager's argument is misleading.
SOLUTION *It is only when $\alpha_p > 0$ that the portfolio manager can beat the market. A simple way of achieving a higher expected return than the market portfolio (i.e., $E(\tilde{r}_p) > E(\tilde{r}_M)$) even when $\alpha_p < 0$, could be to target a portfolio with $\beta_p > 1$, for example by leveraging the portfolio (borrowing to invest in stocks).*

(b) The portfolio manager proposes to pursue a well-known trading strategy (i.e., a portfolio P) of going long in stocks which pay large dividends (relative to the price of the stock) and short in stocks paying little dividends. Suppose you observe that over time the expected return on this portfolio strategy is $E\{\tilde{r}_P\} = 11\%$ while the covariance with $\tilde{r}_M$ is $cov(\tilde{r}_P, \tilde{r}_M) = 0.018$.

i. Based on this observation, what would you conclude about the asset pricing formula in (1.d) and the theory associated with it? Is this a valid empirical test?
SOLUTION *This exercise requires some additional facts. Consider the security market line in the economy above, i.e., $E\{\tilde{r}_M\} = 6\%$, $std(\tilde{r}_M) = 10\%$ and $r_f = 1\%$;*

$$\begin{aligned} & r_f + \frac{\sigma_{pM}}{\sigma_M^2}(E(\tilde{r}_M) - r_f) \\ = \quad & 0.01 + \frac{0.018}{(0.1)^2}(0.06 - 0.01) = 10\% \end{aligned}$$

This implies that the portfolio strategy delivering $E\{\tilde{r}_P\} = 11\%$ does lie above the security market line and does, hence, indeed beat the market. If the return on the "market portfolio" $\tilde{r}_M$ does indeed capture the true market portfolio, then this result would indeed represent a rejection of CAPM. However, following Roll's critique, it could be that the measure of the market portfolio return $\tilde{r}_M$ misses some risk and some assets, in which case $\tilde{r}_M$ could be inside the efficiency frontier. In this case CAPM could be true even though one observes $E\{\tilde{r}_P\} = 11\%$ and $cov(\tilde{r}_P, \tilde{r}_M) = 0.018$.

(c) Given the insight in 2.b, how would you design a plan for evaluating the portfolio manager?

SOLUTION *The way to go is to adopt an Arbitrage Pricing Theory approach and estimate a factor regression including the dividend factor. Illustrate how to run this regression. If the portfolio manager still has a positive intercept in the ATP regression, the portfolio manager can be said to deliver positive risk-adjusted returns.*

3. **Options [15%]**

   (a) Show that one can construct the payoffs of a put option by making a portfolio combining three assets: a call option, riskfree debt, and the underlying asset.

   SOLUTION *Assume underlying share with certainty pays no dividends between now and expiration date of options. Consider following set of four transactions:*

		At expiration	
	Now	If $S_T \leq K$	If $S_T > K$
Sell call option	c	0	$K - S_T$
Buy put option	$-p$	$K - S_T$	0
Buy share	$-S$	S_T	S_T
Borrow (risk free)	Ke^{-rT}	$-K$	$-K$
Total	$c - p - S + Ke^{-rT}$	0	0

   A put option gives payoff $K - S_T$ if $K \leq S_T$ and 0 otherwise. This is equivalent to a portfolio of one call option (with same strike price K as put option), shorting the share, and long in bonds Ke^{-rT}.

   (b) An "American" option can be executed any day until the expiration date. Explain why it can never be optional to execute a put option before the expiration date. Why does this not necessarily hold for call options?

   SOLUTION *American call option on shares which certainly will not pay dividends before option's expiration, should not be exercised before expiration, since*

$$C \geq S - Ke^{-rT} > S - K.$$

   Worth more "alive than dead." When no dividends: Value of American call equal to value of European, since it is not rational to exercise these options early. This does not necessarily hold for call options because a dividend lowers the price of the asset (ex dividend) and if the dividend is sufficiently large, it can be optimal to exercise it just before the dividend date. For a put option, the realization of a dividend will increase the value of the option. Therefore, it follows immediately that a put option will never be realized before the expiration date.

   (c) Suppose the price of a Yara stock is NOK 200. Moreover there exists a call option on Yara with strike price 250 (with strike date in one month). Moreover, assume that there are no dividends over the next month and that the risk free interest is zero. What would be the price of a put option on Yara with strike price 250 in one month?

SOLUTION *Using the above table, absence of arbitrage implies that* $0 = c - p - S + Ke^{-rT}$. *Thus, the price of the put option, p, must be*

$$p = c - S + Ke^{-rT} = c - 200 + 250 * 1$$

(d) Consider a stock with current price $S = 100$. One month from now the stock can either increase by 48% with some probability p or decline by 13.4% with probability $1 - p$. Suppose there exists a call option on the stock with strike price $K = 105$ and strike date one month from now. Moreover, suppose risk free debt has a (compounded) interest rate 1% per month and assume the stock will not have any dividends over the next month.

i. Show that by using a portfolio comprising the bond and the stock it is possible to mimic the payoff of the option.
SOLUTION *Choose stocks* Δ *and bonds* B *so as to satisfy the following equations:*

$$\begin{aligned} u * S\Delta + \exp(r) B &= uS - K \\ d * S\Delta + \exp(r) B &= 0 \end{aligned}$$

ii. What weights does one need to mimic the option?
SOLUTION *Solving the above equation yields*

$$\begin{aligned} B &= -\frac{(Su - K) d}{(u - d) e^r} = -\frac{(100 * 1.48 - 105)(1 - .134)}{(1.48 - (1 - .134)) e^{0.01}} = -60 \\ \Delta &= \frac{Su - K}{S(u - d)} = \frac{100 * 1.48 - 105}{100(1.48 - (1 - .134))} = 0.7 \end{aligned}$$

iii. What is the price of the option?
SOLUTION: *by absence-of-arbitrage, the price of the option c must be*

$$c = S\Delta + B = 100 * 0.7 - 60 = 10$$