

# ECON4910 Environmental Economics — Seminar 4

March 4, 2014

## Problem 1

Assume fossil energy ( $x$ ) and non-fossil energy ( $y$ ) are perfect substitutes as inputs in an aggregate production function. All other inputs are given. Both energy sources have increasing marginal production costs. Fossil energy is assumed to have a negative environmental (climate) impact, while non-fossil energy is assumed to have no environmental impact.

Consider the following alternative policy instruments: (i) a tax on  $x$ , (ii) a subsidy on  $y$ , (iii) a renewable portfolio standard (RPS), i.e. a requirement that  $y \geq \alpha x$ , where  $\alpha$  is a policy parameter

- (a) Derive the optimal tax when (i) is the only policy used, and the optimal subsidy when only (ii) is used.
- (b) Rank the three policies with respect to social welfare when in each case the policy is set so that a specific target  $x^*$  for  $x$  is achieved (same  $x^*$  for all three cases).
- (c) Assume that  $y$  is produced by many small producers, and that future production costs of  $y$  are lower the higher current aggregate production of  $y$  is (due to “learning by doing”). Would this change your ranking of the three policies?
- (d) Assume now that  $y$  is bioenergy, and that production of 1 unit of  $y$  has a negative impact on the environment equal to the impact that  $\gamma$  units of  $x$  has, where  $\gamma$  is an exogenous parameter between 0 and 1. (The climate damage function is in other words now instead of the previous function  $D(x)$ .) Assume that only policy (ii) is used. What can you say about the optimal tax/subsidy on  $y$  in this case?

## Problem 2

There are two sources of energy that are perfect substitutes. At time  $t$  “brown” energy is produced in a quantity  $x(t)$ , giving total costs  $bx(t)$  where  $b$  is fixed and positive. Production of the brown energy gives carbon emissions that lead to an increased amount of carbon in the atmosphere. The stock of carbon in the atmosphere at time  $t$  is denoted  $A(t)$ , and develops over time according to

$$\dot{A}(t) = x(t)$$

where  $\dot{A}(t)$  denotes the derivative of  $A(t)$  with respect to time, and where units are chosen so that one unit of brown energy gives one unit of carbon emissions.

At time  $t$  “green” energy is produced in a quantity  $y(t)$ , giving total costs  $\frac{g}{2}(y(t))^2$  where  $g$  is fixed and positive. Production of green energy has no negative impacts on the environment.

Total energy demand is constant and exogenously given equal to  $E$ , so that  $x(t) + y(t) = E$  for all  $t$ . Assume that  $gE > b$ .

- (a) Find the socially optimal values of  $x(t)$  and  $y(t)$  in the absence of any concern for the environment.
- (b) Assume that energy markets are competitive. Show that for an emission tax  $q(t) < gE - b$ , carbon emissions are given by

$$x(t) = E - \frac{b + q(t)}{g}$$

- (c) Assume that environmental damages are zero for  $A(t) < \bar{A}$  and  $A(t)$  is constrained not to exceed  $\bar{A}$ , where  $\bar{A}$  is exogenous. Show that the socially optimal carbon tax under these assumption can be written as

$$q(t) = q(0)e^{rt}$$

where  $r$  is the interest rate, and explain how  $q(0)$  is determined.

- (d) How is  $q(0)$  affected by a reduction in  $\bar{A}$ ?
- (e) How is  $q(0)$  affected by a reduction in  $g$ ?
- (f) Instead of the assumptions made in (c) above, let us now assume that environmental damages at time  $t$  are equal to  $vA(t)$  for  $A(t) < \bar{A}$  and that we as before do not permit  $A(t)$  to exceed  $\bar{A}$ . How do the properties of the optimal carbon tax depend on the exogenous parameter  $v$ ?