Seminar 6 – Renewable Resources

Solutions

Question 1: Fishery-1

Assume that the stock dynamics are accurately described by a logistic growth function and the harvest production function is of the Gordon-Schaefer type $H = ES$ (where H is harvest, E is effort (suitably scaled), and S is the fish stock).

1-1. *Can the fish stock be harvested to extinction under open access? Could it be optimal to harvest it to extinction?*

Presume that – as in the lecture – the price of landed harvest is p and the unit cost of effort is w. Then, since open-access results in zero profits in equilibrium, we have:

$$\begin{aligned} pH^{OA} &= wE^{OA} \\ pE^{OA}S^{OA} &= wE^{OA} \\ S^{OA} &= \frac{w}{p} \end{aligned}$$

So that the open-access stock will be declining with the cost-price ratio, but it will be exhausted if and only if the effort costs are zero. Since the growth function is logistic, there is no threshold below which the stock would not recover from harvesting. Loosely speaking, the stock-dependency of the harvesting cost $c(H, S) = w\frac{H}{S}$ makes it excessively expensive to catch the last fish in the ocean.

The short (and fully sufficient) answer to the optimal exhaustion question is "no, unless $w = 0$". From class we know that for an optimal exhaustion both the discount rate must be higher than the maximum growth rate of the stock, and it must be profitable to harvest the last unit of the resource. The latter condition is not fulfilled for $w > 0$ as $\lim_{S\to 0} c(H, S) = \lim_{S\to 0} w\frac{H}{S} = \infty$. An equally valid way of reasoning is to note that the stock will under optimal management be at least as high as under open access. It is positive (with the caveat that $w > 0$) under open access and must hence also be positive under optimal management.

For the long answer according to the book, set up the optimal control problem as in the lecture notes. In equilibrium, we are then left with three equations:

$$H^* = G(S^*) \tag{1}$$

$$\mu = p - c_H(H^*, S^*) \tag{2}$$

$$r = G'(S^*) - \frac{c_S(H^*, S^*)}{\mu} \tag{3}$$

and with our specific functions, this can be written as two equations in two unknowns (H^* and S^*):

$$H^* = gS^*\left(1 - \frac{S^*}{S_{max}}\right)$$

$$p - \frac{w}{S^*} = \frac{wH^*}{S^*\left(r - g\left(1 - \frac{2S^*}{S_{max}}\right)\right)}$$

this can be solved for the optimal steady state stock (compare Appendix 17.3 in Perman et al.):

$$S^* = \frac{S_{max}}{4}\left(1 - \frac{r}{g} + \frac{w}{pS_{max}} + \sqrt{\left(1 - \frac{r}{g} + \frac{w}{pS_{max}}\right)^2 + \frac{8wr}{pgS_{max}}}\right)$$

This rather unintelligible equation doesn't give us a lot of intuition. It is clear though that if $w = 0$ and $r > g$ we indeed get a non-positive S^*.

Now consider the case where the harvest function is independent of stock size $H = E$

1-2. *Under which circumstances could such a harvest function be a good approximation?*

This a situation where the density of fish remains unchanged, but the area occupied by the fish declines as the stock gets depleted. This could be a reasonable approximation for schooling species, where each school can be located at low cost (e.g. by use of sonar devices).

1-3. *Will the fish stock now be harvested to extinction under open access? Could it be optimal to harvest it to extinction?*

As harvesting costs are independent of stock size $c(H, S) = wH$ we have that under open-access, the stock will be exhausted whenever fishing is profitable ($p > w$).

For the optimal solution, we see that equation (3) simplifies to:

$$
\begin{aligned}
r &= G'(S^*) \\
r &= g\left(1 - \frac{2S^*}{S_{max}}\right) \\
S^* &= \frac{S_{max}}{2}\left(1 - \frac{r}{g}\right).
\end{aligned}
$$

In other words, in the case where costs do not depend on stock size, the resource stock is maintained at a level where the rate of biological growth equals the market rate of return on investment. Resource exhaustion will be optimal whenever $r > g$.

Question 2: Fishery-2

The answers can also be found in the "assessment guidance" for the 2007 exam: `http://www.oekonomi.uio.no/studier/eksamen/tidligere%20eksamensoppgaver/sensorveiledninger/master/4925/4925.h07.pdf`

A stock of fish develops dynamically as follows:

$$
\dot{S}(t) = gS\left(1 - \frac{S(t)}{2}\right) - H(t) \tag{4}
$$

where S is the stock of fish (and $\dot{S}$ is the change per year), H is the rate of harvest per year, and g is a positive parameter.

2-1. *Give a short critical discussion of the assumptions underlying equation* (4)*.*
This is a simple aggregate biomass model, which neither addresses environmental fluctuations/stochasticity, multi-species/ecosystem aspects nor the internal structure of the fish stock. Most importantly, it implies that the fish stock will grow again to its maximal size from any positive initial value (no matter how small) when harvesting is suspended. Yet, the model is fully sufficient as a pedagogical to explain the impact of different management institutions.

2-2. *What is the maximal possible stock of the fish?*
If harvesting is suspended ($H(t) = 0$) the stock will grow to its maximal size which is $S_{max} = 2$. `Here, you write` "And this makes it a plausible assumption that $S_{max} = 2$."

2-3. *What is the maximal sustainable harvest rate, and what is the corresponding stock of fish?*

Equation (4) describes the standard logistic model with $S_{max} = 2$, so that one could jump straight to the conclusion that $S_{MSY} = 1$ and correspondingly $H_{MSY} = G(S_{MSY}) = g/2$. More explicitly, sustainable harvesting implies $\dot{S} = 0$ and $S > 0$ so that equation (4) can be written as:

$$H = gS\left(1 - \frac{S}{2}\right) \ = \ gS - \frac{g}{2}S^2$$

which is maximized for $S = 1$ and hence $H = \frac{g}{2}$.

The production function for this fishing industry is given by $H = E^{\alpha}S^{1-\alpha}$ where E is aggregate effort and $0 < \alpha < 1$. The price of effort is w and the price of fish is p.

2-4. *What is the long-run stock of fish in an unregulated open-access fishery?*

First of all, it is useful to express cost in terms of harvest and use the production function $H(E,S) = E^{\alpha}S^{1-\alpha} \Rightarrow E(H,S) = H^{1/\alpha}S^{(\alpha-1)/\alpha}$ to arrive at $c(H,S) = wH^{1/\alpha}S^{(\alpha-1)/\alpha}$

Under open-access, there will be no profits in equilibrium:

$$pH = wE$$

which, upon using the reformulation from above, is equal to:

$$pH = w\left(\frac{H}{S^{1-\alpha}}\right)^{\frac{1}{\alpha}}$$
$$\Leftrightarrow$$
$$H = \underbrace{\left(\frac{p}{w}\right)^{\frac{\alpha}{1-\alpha}}}_{A^{OA}} S \ = \ A^{OA}S \tag{5}$$

Then, inserting this into (4) gives:

$$\begin{aligned}\dot{S} &= gS\left(1 - \frac{S}{2}\right) - A^{OA}S \\ &= (g - A^{OA})S\left(1 - \frac{S}{2}\right)\end{aligned}$$

Hence, the stock will decline to zero when $g < A^{OA}$. When $g > A^{OA}$, the stock will approach a positive steady state which is given by:

$$S^{OA} = 2\left(1 - \frac{A^{OA}}{g}\right) \tag{6}$$

2-5. *Derive the conditions for the socially optimal harvest, and show that the long-run stock of fish in this case, if it is positive, will be higher than in the unregulated open-access fishery.*

Note that this questions asked you to "derive" the conditions for socially optimal harvest, but it was not stated what socially optimal is. Therefore assume that this means maximizing the net-present-value of the fishery (the sum of discounted revenue minus costs), or in other words:

$$\max_H \Pi \int_0^\infty [pH - c(H,S)]e^{-rt}\mathrm{d}t \tag{7}$$
$$\text{subject to:} \quad \dot{S} = G(S) - H; \quad S(0) = S_0, S \geq 0; \quad 0 \leq H \leq H_{max}$$

The current value Hamiltonian is then:

$$\mathcal{H} = pH - c(H,S) + \mu(G(S) - H), \tag{8}$$

and a necessary conditions for optimality is:

$$p - c_H(H,S) - \mu \leq 0 \quad (= 0 \text{ for } H > 0), \tag{9}$$

We omit a discussion of the approach dynamics and the "Hotelling rule for renewable resources" here. Rather, we seek to express harvest and stock size in terms of the parameters. Further, from the interpretation of μ , we know that it is positive when there is a positive stock. For an interior solution, we therefore have:

$$\begin{aligned}
c_H(H,S) &= p - \mu \\
&\Rightarrow \\
w\frac{1}{\alpha}H^{\frac{1-\alpha}{\alpha}}S^{\frac{\alpha-1}{\alpha}} &= p - \mu \\
&\Leftrightarrow \\
H^{\frac{1-\alpha}{\alpha}} &= \frac{\alpha(p-\mu)}{w}S^{\frac{1-\alpha}{\alpha}} \\
&\Leftrightarrow \\
H &= \underbrace{\left(\frac{\alpha(p-\mu)}{w}\right)^{\frac{\alpha}{1-\alpha}}}_{A^*} S \ = \ A^* S
\end{aligned} \tag{10}$$

Then, inserting this into (4) gives:

$$\dot{S} = gS\left(1 - \frac{S}{2}\right) - A^* S$$
$$= (g - A^*)S\left(1 - \frac{S}{2}\right)$$

Hence, the stock will decline to zero when $g < A^*$. When $g > A^*$, the stock will approach a positive steady state which is given by:

$$S^* = 2\left(1 - \frac{A^*}{g}\right) \tag{11}$$

Then, as $\mu > 0$ and $0 < \alpha < 1$ we know that $A^* < A^{OA}$ and consequently $S^* > S^{OA}$.

2-6. *Show how a regulator can design a policy so that the long-run outcome of the open-access fishery becomes identical to the long-run social optimum.*

Having derived (5) and (10), it is plain to see that a Pigouvian tax $\tau = \alpha\mu + (1-\alpha)p$ will bring A^{OA} to be of the same size as A^*, where $\alpha\mu$ corrects for the fact that fishers ignore the future and $(1-\alpha)p$ corrects the crowding externality.

Exkurs: Equivalence of pigouvian tax and marketable ITQs

Let the profit function of the individual fisherman be given by:

$$\pi = pH_i - c(H_i, S) \quad \text{subject to: } H_i \leq Q_i$$

where Q_i is the quota endowment of agent i. Suppose there exists a well-functioning market for quotas with market price per unit of quota given by m. Then, the agent would want to buy additional quotas whenever $\partial\pi/\partial Q_i > m$ and he would want to sell quotas whenever $\partial\pi/\partial Q_i < m$, so that $\partial\pi/\partial Q_i = m$ specifies the demand function $D_i(m,S) = Q_i$ of the i's fisherman. The aggregate demand $D(m,S)$ must then equal the supply of quotas Q_T. For a given level of S, it is a decreasing function of m with a unique solution of $D(m,S) = Q_T$, so that, theoretically, by setting the appropriate total quota level, the stock can be steered to the optimal steady state, and any market price can be achieved, in particular the quota market price $m^* = \tau = \alpha\mu + (1-\alpha)p$.

Question 3: Forestry

The answers to questions **3-1.** to **3-3.** can also be found here: `http://personal.strath.ac.uk/r.perman/qanda18.htm`

3-1. *Derive the optimal rotation length in an infinite horizon model.*

In the infinite horizon plantation forestry model, profits are given by the infinite sequence:

$$\max_T \Pi = px(T)e^{-rT} - k + e^{-rT}[px(T)e^{-rT} - k] + e^{-2rT}[px(T)e^{-rT} - k] + \dots$$

$$= px(T)e^{-rT} - k + e^{-rT}\Pi$$

$$= \frac{px(T)e^{-rT} - k}{1 - e^{-rT}}$$

which can be manipulated so that T appears only once in the numerator:

$$\frac{px(T) - k}{e^{rT} - 1} - k \qquad (*)$$

The crucial assumption is that nothing changes over time (prices, external conditions) and that therefore, the rotation period T will be the same in all sequences. The value of T that maximizes the expression in (*) is then found by the following first-order-condition:

$$\frac{\partial \Pi}{\partial T} = \frac{px'(T)(e^{rT} - 1) - (px(T) - k)re^{rT}}{(e^{rT} - 1)^2} = 0$$

$$\Rightarrow$$

$$\frac{px'(T)}{px(T) - k} = \frac{r}{1 - e^{-rT}}$$

$$\Leftrightarrow$$

$$px'(T) = rpx(T) + r\Pi \qquad (**)$$

where the LHS of (**) is the value growth when the timber is left standing for an marginal unit of time and the RHS consist of the corresponding cost of doing so: first, the money lost from not putting the returns from the timber harvest in the bank, and second the money lost from not starting a new cycle (and exploiting the fast growth early on) or selling the plot at its opportunity price (this second term is called the site value or land value).

3-2. *Show and provide intuition for changes in the optimal rotation period due to an increase in*

In order to do "show" the results for this comparative statics exercise, we need to derive the total differential. The idea here is that equation (**) is a first-order-condition of the form $f(k,c,P,r,\Pi) = 0$ and we are interested in how the optimal value of T changes when one of the parameters[1] k,c,P,r,Π changes. As we are interested in the optimal T $f(k,c,P,r,\Pi) = 0$ still has to hold. The total differential wrt to some parameter x is then given by: $\frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial T}\mathrm{d}T = 0$ and the rotation period increases when $\frac{\mathrm{d}T}{\mathrm{d}x}$ is positive.

Now, for future reference, re-write (**) as

$$0 = x'(T) - rx(T) - \frac{r}{p}\Pi$$

and $\partial f \partial T = x''(T) - rx'(T)$ is then negative (provided the growth function $x(T)$ is concave in the region of interest, but this should be the case). The optimal rotation period will then increase when x increases as long as $\frac{\partial f}{\partial x}$ is *negative*. Let x be:

(a) *planting cost, then:* $k \nearrow; T \nearrow$
$\frac{\partial f}{\partial k} = -\frac{r}{p}\frac{\partial \Pi}{\partial k} = -\frac{r}{p}\frac{e^{rT}}{e^{rT}-1} < 0$, so that an increase in planting cost increases the rotation interval. The intuition is that increased planting cost lower the profitability of the forestry and hence decreases the opportunity cost of delaying replanting.

(b) *harvesting cost, then:* $c \nearrow; T \nearrow$

(c) *the gross price of timber, then:* $P \nearrow; T \searrow$
$\frac{\partial f}{\partial p} = \frac{r}{p^2}\Pi - \frac{r}{p}\frac{\partial \Pi}{\partial p} = \frac{r}{p}\left(\frac{-px(T)+e^{rT}k}{p(e^{rT}-1)} + \frac{x(T)}{e^{rT}-1}\right) = \frac{rk}{p}\left(\frac{e^{rT}}{e^{rT}-1}\right) > 0$ As the rotation interval decreases with increased *net* prices, it follows that the rotation interval increases with increased costs c and decreases with increased gross prices P, according to the changes in the value of the land.

(d) *the discount rate, then:* $r \nearrow; T \searrow$
$\frac{\partial f}{\partial r} = x(T) - \frac{1}{p}\Pi + \frac{r}{p}\frac{\partial \Pi}{\partial r}$
$= x(T) - \frac{1}{p}\Pi + \frac{r}{p}\left(-\frac{px(T)Te^{-rT}(1-e^{-rT}) - (px(T)e^{-rT}-k)Te^{-rT}}{(1-e^{-rT})^2}\right) = \left(\frac{\Pi}{p} + x(T)\right)\left[1 - \frac{rT}{e^{rT}-1}\right]$ Where the term in the brackets is positive. Intuitively, the more impatient I am the more often I would want to restart my growing cycle to reap the fast growth early on.

(e) *the productivity of agricultural land*
I interpret an increase in the productivity of agricultural land as an increase in the opportunity cost of the planting site and hence an increase in land value. As we have discussed many times now, an increase in the value of the land decreases the rotation period. Perman et al in their answers interpret "an increase in the productivity of agricultural land" as an increased value in the outside option, and model this as an increase in the discount rate, yielding the same result that $T \nearrow$.

[1] Note that the land value Π is here implicitly taken to be a parameter.

3-3. *Demonstrate that a tax τ imposed on each unit of timber felled will increase the optimal rotation length.*

A tax τ on each unit of timber felled implies that the net price decreases: $\tilde{p} = P - c - \tau < P - c = p$. Hence it works as an increase in harvesting cost to lengthen the rotation period.

3-4. *Discuss the impact on the rotation length if in addition a subsidy of the same magnitude τ were paid for timber growth.*

When a subsidy of the same magnitude is paid for timber growth, the value of this tax/subsidy scheme will be positive over the rotation period. Intuitively, that makes sense: for every period that there is some stock, the forest owner gets paid. Only in the period when she cuts down the trees she has to pay. So in the period immediately before she cuts, she gets paid almost as much as she has to repay the period after – plus, she got paid all the way long. Mathematically, this can be shown by writing the value of the tax/subsidy scheme as:

$$N(T) = \int_0^T s\dot{x}(t)e^{-rt}\mathrm{d}t \quad - sx(T)e^{-rT}$$

To show that it is positive integrate by parts (recall: $\int_a^b f(x)g'(x)dx = f(x)g(x) - \int_a^b f'(x)g(x)dx$, here choose: $f(x) = e^{-rt}$ and $g'(x) = s\dot{x}(t)$) and make use of the fact that $x(0) = 0$:

$$\begin{aligned} N(T) &= \left[x(t)e^{-rt}\right]_0^T + \int_0^T rsx(t)e^{-rt}\mathrm{d}t - sx(T)e^{-rT} \\ &= sx(T)e^{-rT} + \int_0^T rsx(t)e^{-rt}\mathrm{d}t - sx(T)e^{-rT} \\ &= \int_0^T rsx(t)e^{-rt}\mathrm{d}t > 0 \end{aligned}$$

additionally we have:

$$N'(T) = rsx(T)e^{-rT} > 0$$

The book (Perman et al 2005, p.614) then says that the rotation period will be longer when the non-timber benefit are increasing with age. This is the case here. However, the following calculations[2] show that rotation length is decreasing:

The value of land is the $rV(\hat{T})$ in Faustmann's rule ($\dot{x}(\hat{T}) = r[x(\hat{T}) + V(\hat{T})]$). The interpretation is that it represents the opportunity cost of land, i.e. what the land is worth when you can plant trees on it forever and after.

To find what happens to the value of land, let us first look at the optimization problem

[2] I copy these calculation's from last year's seminar where the students were given that $\frac{N'(T)}{N(T)} > \frac{re^{-rT}}{(1-e^{-rT})}$.

the without the tax/subsidy scheme: (assuming $p = 1$ and neglecting cutting costs k):

$$V(\hat{T}) = x(\hat{T})e^{-r\hat{T}} + V(\hat{T})e^{r\hat{T}}$$

Solving for $V(T)$:

$$\hat{V}(\hat{T}) = \frac{x(\hat{T})e^{-r\hat{T}}}{1 - e^{-r\hat{T}}}$$

The optimization problem taking the tax/subsidy scheme into consideration is:

$$\begin{aligned} V(T) &= x(T)e^{-rT} + N(T) + V(T)e^{-rT} \\ V(T) &= \frac{x(T)e^{-rT} + N(T)}{1 - e^{-rT}} \end{aligned} \tag{12}$$

Above we have shown that $N(T) > 0$. Hence we see here that for *any* T, $V > \hat{V}$. Hence, the value of land increases. Suppose the forest owner does not change a thing (keeps the formerly optimal cutting time): then she will be better off. But she can do even better: Define $x(T)e^{-rT} = v(T)$. The optimal cutting time is found by optimizing the objective function found in (12). By differentiating we get the following first order condition:

$$\tilde{V}'(T) = \frac{(1 - e^{-rT})[v'(T) + N'(T)] - re^{-rT}[v(T) + N(T)]}{(1 - e^{-rT})^2} = 0$$

Simplyfying some:

$$\begin{aligned} v'(T) + N'(T) &= \frac{re^{-rT}[v(T) + N(T)]}{(1 - e^{-rT})} \\ \frac{v'(T) + N'(T)}{v(T) + N(T)} &= \frac{re^{-rT}}{(1 - e^{-rT})} \end{aligned}$$

Now, we are given that:

$$\frac{N'(T)}{N(T)} > \frac{re^{-rT}}{(1 - e^{-rT})}$$

Then we can write:

$$\begin{aligned}
\frac{N'(T)}{N(T)} &> \frac{v'(T)+N'(T)}{v(T)+N(T)} \\
N'(T)[v(T)+N(T)] &> [v'(T)+N'(T)]N(T) \\
N'(T)v(T) &> v'(T)N(T) \\
\frac{N'(T)}{N(T)} &> \frac{v'(T)}{v(T)}
\end{aligned}$$

Now we wish to show that:

$$\begin{aligned}
\frac{v'(T)}{v(T)} &\overset{!}{<} \frac{v'(T)+N'(T)}{v(T)+N(T)} \qquad \text{(x)} \\
v'(T)[v(T)+N(T)] &< [v'(T)+N'(T)]v(T) \\
v'(T)N(T) &< N'(T)v(T) \\
\frac{v'(T)}{v(T)} &< \frac{N'(T)}{N(T)}
\end{aligned}$$

So the inequality holds, since we have just above shown that the latter condition holds. So with no tax/subsidy scheme we had the following optimality condition:

$$\frac{v'(\hat{T})}{v(\hat{T})} = \frac{re^{-r\hat{T}}}{(1-e^{-r\hat{T}})}$$

From the equation (x) we get that:

$$\frac{re^{-r\hat{T}}}{(1-e^{-r\hat{T}})} < \frac{re^{-rT}}{(1-e^{-rT})}$$

Since the function is decreasing in T, that means $T < \hat{T}$. The tax/subsidy scheme implies shorter rotation periods.